

```

SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-12"];
<< FreeLie.m;
BCH[n_Integer] := BCH[{"x"}, {"y"}][n];
BCHWords[n_Integer] := Cases[BCH[n], _LW, Infinity];
AllLyndonWords[n_Integer] := AllLyndonWords[n, {"x", "y"}];
    
```

BCH[7]

$$\begin{aligned}
 & \frac{\langle xxxxxxxy \rangle}{30\,240} - \frac{\langle xxxxxxxyy \rangle}{5040} + \frac{\langle xxxxyxyx \rangle}{10\,080} + \frac{\langle xxxxyyyy \rangle}{3780} + \frac{\langle xxxyxxyx \rangle}{10\,080} + \frac{\langle xxxyxxyy \rangle}{1680} + \\
 & \frac{\langle xxxxyxyx \rangle}{1260} + \frac{\langle xxxxyyyy \rangle}{3780} + \frac{\langle xxyxxyy \rangle}{2016} - \frac{\langle xxyxyxy \rangle}{5040} + \frac{13 \langle xxyxyyy \rangle}{15\,120} + \frac{\langle xxyyxyy \rangle}{10\,080} - \\
 & \frac{\langle xxyyyyxy \rangle}{1512} - \frac{\langle xxyyyyxy \rangle}{5040} + \frac{\langle xyxyxyy \rangle}{1260} - \frac{\langle xyxyyyy \rangle}{2016} - \frac{\langle xyxyyyy \rangle}{5040} + \frac{\langle xyyyyxy \rangle}{30\,240}
 \end{aligned}$$

Expand[2 BCH[8]]

$$\begin{aligned}
 & \frac{\langle xxxxxxxyy \rangle}{30\,240} - \frac{\langle xxxxxxxyxy \rangle}{7560} - \frac{\langle xxxxxxxyyy \rangle}{5040} + \frac{\langle xxxxyxxyx \rangle}{10\,080} - \frac{\langle xxxxyxxyy \rangle}{10\,080} + \frac{\langle xxxxyxyxy \rangle}{1260} + \\
 & \frac{23 \langle xxxxyyyy \rangle}{60\,480} + \frac{\langle xxxyxxyy \rangle}{2016} - \frac{\langle xxxxyxyxy \rangle}{5040} + \frac{13 \langle xxxxyxyyy \rangle}{15\,120} + \frac{\langle xxxxyxyxy \rangle}{10\,080} - \\
 & \frac{\langle xxxxyxyxy \rangle}{1512} - \frac{\langle xxxxyyyy \rangle}{5040} + \frac{\langle xxyxyxyy \rangle}{1260} - \frac{\langle xxyxyyyy \rangle}{2016} - \frac{\langle xxyyxyyy \rangle}{5040} + \frac{\langle xxyyyyxy \rangle}{30\,240}
 \end{aligned}$$

2 BCH[8] - b[{"x"}, BCH[7]] // Expand

$$-\frac{\langle xxxxxxxyy \rangle}{30\,240} + \frac{\langle xxxxxxxyy \rangle}{4320} - \frac{\langle xxxxxxxyxy \rangle}{4320} - \frac{\langle xxxxxxxyyy \rangle}{2160} - \frac{\langle xxxxyxxyy \rangle}{1440} + \frac{\langle xxxxyxyxy \rangle}{8640}$$

b[{"y"}, BCH[7]] // Expand

$$\begin{aligned}
 & -\frac{\langle xxxxxxxyy \rangle}{30\,240} + \frac{\langle xxxxxxxyxy \rangle}{7560} + \frac{\langle xxxxxxxyyy \rangle}{5040} - \frac{\langle xxxxyxxyx \rangle}{10\,080} + \frac{\langle xxxxyxxyy \rangle}{10\,080} - \\
 & \frac{\langle xxxxyxyxy \rangle}{1260} - \frac{\langle xxxxyyyy \rangle}{3780} - \frac{\langle xxxxyxxyy \rangle}{2016} + \frac{\langle xxxxyxyxy \rangle}{5040} - \frac{13 \langle xxxxyxyyy \rangle}{15\,120} - \\
 & \frac{\langle xxxxyxyxy \rangle}{1260} - \frac{\langle xxxxyxyxy \rangle}{3780} - \frac{\langle xxxxyyyy \rangle}{3780} - \frac{\langle xxyxyxyy \rangle}{4320} - \frac{\langle xxyxyxyy \rangle}{10\,080} + \frac{\langle xxyxyxyxy \rangle}{1440} - \\
 & \frac{17 \langle xxyxyyyy \rangle}{15\,120} + \frac{\langle xxyxyxyxy \rangle}{720} - \frac{29 \langle xxyxyxyyy \rangle}{30\,240} + \frac{\langle xxyxyxyxy \rangle}{2160} + \frac{\langle xxyxyxyxy \rangle}{1080} + \\
 & \frac{\langle xxyxyyyy \rangle}{5040} - \frac{\langle xyxyxyyy \rangle}{432} + \frac{\langle xyxyxyxy \rangle}{1440} + \frac{\langle xyxyxyyy \rangle}{1440} + \frac{\langle xyxyxyyy \rangle}{1440} - \frac{\langle xyxyxyyy \rangle}{30\,240}
 \end{aligned}$$

2 BCH[10] - b["x"], BCH[9] // Expand

$$\frac{\langle \text{xxxxxxxxxy} \rangle}{1209600} - \frac{\langle \text{xxxxxxxxxyy} \rangle}{134400} + \frac{\langle \text{xxxxxxxxxyx} \rangle}{67200} + \frac{11 \langle \text{xxxxxxxxyyy} \rangle}{453600} - \frac{\langle \text{xxxxxxxxyxy} \rangle}{120960} + \frac{\langle \text{xxxxxxxxxyyy} \rangle}{50400} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{15120} - \frac{61 \langle \text{xxxxxxxxyyy} \rangle}{1814400} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{24192} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{60480} - \frac{13 \langle \text{xxxxxxxxyyy} \rangle}{181440} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{120960} + \frac{\langle \text{xxxxxxxxyyyxy} \rangle}{18144} + \frac{\langle \text{xxxxxxxxyyyyy} \rangle}{60480} - \frac{\langle \text{xxxxxxxxyxyyy} \rangle}{15120} + \frac{\langle \text{xxxxxxxxyxyyy} \rangle}{24192} + \frac{\langle \text{xxxxxxxxyxyyy} \rangle}{60480} - \frac{\langle \text{xxxxxxxxyyyyy} \rangle}{362880}$$

BCH[9]

$$- \frac{\langle \text{xxxxxxxxxy} \rangle}{1209600} + \frac{\langle \text{xxxxxxxxxyy} \rangle}{151200} - \frac{\langle \text{xxxxxxxxxyx} \rangle}{100800} - \frac{\langle \text{xxxxxxxxyyy} \rangle}{56700} - \frac{\langle \text{xxxxxxxxyxy} \rangle}{43200} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{75600} + \frac{\langle \text{xxxxxxxxyyy} \rangle}{75600} + \frac{\langle \text{xxxxxxxxyxxx} \rangle}{302400} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{67200} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{43200} + \frac{11 \langle \text{xxxxxxxxyyy} \rangle}{302400} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{25200} + \frac{11 \langle \text{xxxxxxxxyxyy} \rangle}{201600} + \frac{11 \langle \text{xxxxxxxxyyyxy} \rangle}{151200} + \frac{\langle \text{xxxxxxxxyyy} \rangle}{75600} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{43200} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{37800} + \frac{23 \langle \text{xxxxxxxxyyy} \rangle}{302400} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{30240} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{100800} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{33600} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{20160} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{67200} - \frac{17 \langle \text{xxxxxxxxyxyx} \rangle}{100800} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{30240} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{21600} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{15120} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{7560} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{56700} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{25200} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{8400} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{17280} - \frac{23 \langle \text{xxxxxxxxyyy} \rangle}{25200} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{50400} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{6048} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{20160} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{10080} - \frac{23 \langle \text{xxxxxxxxyyy} \rangle}{302400} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{60480} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{20160} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{20160} - \frac{11 \langle \text{xxxxxxxxyyy} \rangle}{120960} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{10080} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{90720} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{60480} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{21600} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{151200} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{10080} - \frac{\langle \text{xxxxxxxxyxyy} \rangle}{7560} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{15120} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{15120} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{43200} + \frac{\langle \text{xxxxxxxxyxyx} \rangle}{33600} + \frac{\langle \text{xxxxxxxxyxyy} \rangle}{60480} - \frac{\langle \text{xxxxxxxxyxyx} \rangle}{1209600}$$

b["x"], #] & /@ BCHWords[9]

- {⟨xxxxxxxxxy⟩, ⟨xxxxxxxxxyy⟩, ⟨xxxxxxxxxyx⟩, ⟨xxxxxxxxyyy⟩, ⟨xxxxxxxxyxy⟩,
- ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyyy⟩, ⟨xxxxxxxxyxxx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩,
- ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyyy⟩,
- ⟨xxxxxxxxyxxx⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩,
- ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩,
- ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩,
- ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩,
- ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩,
- ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩,
- ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩,
- ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩,
- ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩, ⟨xxxxxxxxyxyx⟩, ⟨xxxxxxxxyxyy⟩}

```

SymmetricDifference[A_List, B_List] := {
  Complement[A, B], Complement[B, A]
};
Table[
  SymmetricDifference[
    b[<"x">, #] & /@ BCHWords[2 n - 1],
    BCHWords[2 n]
  ],
  {n, 2, 8}
] // ColumnForm

{{<xxx>, {}}
{{<xxxx>, {}}
{{<xxxxx>, {}}
{{<xxxxxxx>, {}}
{{<xxxxxxxx>, {<xxxxxxxxxy>}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {<xxxxxxxxxyxy>}}

```