Quantum topology (easy) | classical topology (hard & mysterious)

Jones polynomial $J_K(q)$:

$$q J_{A_k}(q) - a^{-1} J_{A_k}(q) = (q^{1/2} - q^{-1/2}) J_{A_i}(q)$$

$$J_A(q) = q^{1/2} + q^{-1/2}$$

$$J_K(q) \in \mathbb{Z}[q^{1/2}]$$

Coloured Jones Poly...

Definition: A sequence $f_i(q) \in \mathbb{Q}(q^{1/2})$ is $q$-holonomic if it satisfies a linear recursion relation with coefficients polynomials in $q$ and $q^{-1}$.

--- The A-J conjecture
In 2004, Aganagic-Vafa conjectured that one more variable can be added to this story: “the Homfly variable.”

In 2007, Gukov-Hong conjectured you can add another variable for Khovanov homology.