The Happy Segway Principle

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$$\frac{d}{dt}\frac{t}{\sqrt{1+t^2}} = \frac{1}{\sqrt{1+t^2}} - \frac{t^2}{\sqrt{1+t^2}^{\,3}}$$

$$= \frac{1+t^2-t^2}{(1+t^2)^{3/2}} = \frac{1}{(1+t^2)^{3/2}}$$

Minimize $\displaystyle\int \frac{\sqrt{1+y'^2}}{v(x,y)}\,dx$

E-L: $F_y - \frac{d}{dx}F_{y'}$:

$$-\frac{V_y}{V^2}\sqrt{1+y'^2} - \frac{d}{dx}\frac{y'}{V\sqrt{1+y'^2}} =$$

$$-\frac{V_y}{V^2}\sqrt{1+y'^2} + \frac{V_x}{V^2}\frac{y'}{\sqrt{1+y'^2}} + \frac{V_y}{V^2}\frac{(y')^2}{\sqrt{1+y'^2}} - \frac{y''}{V}\frac{1}{(1+y'^2)^{3/2}}$$

$$= \frac{V_x y' + V_y\left(y'^2 - (1+y'^2)\right)}{V^2\sqrt{1+y'^2}} - \frac{y''}{V}\frac{1}{(1+y'^2)^{3/2}}$$

$$= \frac{1}{V^2(1+y'^2)^{3/2}}\left((y'V_x - V_y)(1+y'^2) - Vy''\right)$$

$$\Longrightarrow \quad Vy'' = (1+y'^2)(y'V_x - V_y)$$