2 LOTs and higher ribbon knots

A labelled oriented tree (LOT) is a tree $\Gamma$, with vertex set $V = V(\Gamma)$, edge set $E = E(\Gamma)$, and initial and terminal vertex maps $i, \tau: E \to V$, together with an additional map $\lambda: E \to V$. For any edge $e$ of $\Gamma$, $\lambda(e)$ is called the label of $e$. In general, one can consider LOTs of any cardinality, but for the purposes of the present paper, every LOT will be assumed to be finite.

To any LOT $\Gamma$ we associate a presentation

$$\mathcal{P} = \mathcal{P}(\Gamma) : \langle V(\Gamma) \mid i(e)\lambda(e) = \lambda(e)\tau(e) \rangle$$

of a group $G = G(\Gamma)$, and hence also a 2-complex $K = K(\Gamma)$ modelled on $\mathcal{P}$. The 2-complex $K$ is a spine of a ribbon disk complement $D^4 \setminus k(D^2)$ [7], that is the complement of an embedded 2-disk in $D^4$, such that the radial function on $D^4$ composed with the embedding $k$ is a Morse function on $D^2$ with no local maximum. Conversely, any ribbon disk complement has a 2-dimensional spine of the form $K(\Gamma)$ for some LOT $\Gamma$.

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"LOT groups & higher ribbon knot groups are precisely the same thing."

Can/should be generalized to forests, "LOT groups".