\[ \theta = p dx + q dy \quad \text{with} \quad p, q \in \mathbb{R}[x, y] \]

or

\[ \dot{x} = q(x, y) \]
\[ \dot{y} = -p(x, y) \]

The \textit{w-limit} of a trajectory is the set of all limits \( \lim_{t \to \infty} x(t) \) for some \( t_n \to \infty \).

\textbf{Poincaré-Bendixson} In the plane an \textit{w-limit} set is either a polycycle:

\begin{itemize}
    \item [\textbullet] sing. pt.
    \item [\textbullet] path between sing. points.
\end{itemize}

or a periodic trajectory; this has two cases:

- integrable
- limit cycle.
Hilbert 6th problem: How many limit cycles may appear in a system of degree n?

This is very hard. (Ilyashenko, Ecalle) Every specific system has finitely many limit cycles.

Even for quadratic systems no uniform bound is known. Currently the largest number known is 4.