Define $m_{F(n)}$ as the unique linear map from $F^{(n)} \otimes F^{(n)}$ to $F^{(n)}$, such that for any $(s, t, \phi)$ and $(s', t', \phi')$ in $P_n$, we have

$$m_{F(n)}(z(s, t, \phi) \otimes z(s', t', \phi')) = \sum_{(c_1, c'_1) \in M_{t_1, s'_1}, \ldots, (c_n, c'_n) \in M_{t_n, s'_n}} \cdots$$

A construction analogous to that of Appendix B shows the following:

**Proposition C.1.** $(F^{(n)}, m_{F(n)})$ is an associative algebra.