Bridson on "From Groups, Curvature"

Was "Groups that Want to be Free"

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* Let negative curvature arise from core group theory.
* Solve some classical hard problems.

"Triangles are uniformly thin"

"Linear Isoperimetric Inequality":

$$\text{Area} \leq \text{cons} \cdot \text{perimeter}$$

"CAT(0)": Triangles are no fatter than in Euclidean space.

$$G = \langle a_1 \ldots a_n \mid r_1 \ldots r_m \rangle$$

In general, impossible to understand. We try to understand "generic" groups.

.... turning group presentations into topological spaces. ... In fact, coverings on which the group acts by deck transformations.

... embed in $\mathbb{R}^5$, thicken, take boundary, and find that every f.p. group is the π_1 of a 4-manifold.

The "word metric" on groups.
Drawing the free group in hyperbolic space.

Gromov: On a group, having thin triangles is equivalent to having a linear isoperimetric innequality.

Gromov: "Random groups are hyperbolic".

Groups that want to be free.

What is freer than a hyperbolic group?

(1) 1st order logic: Which group think that they are free from the perspective of first order logic?

Th(Γ) = "The set of all first order sentences true in Γ"

Example

$$\exists t\ \forall x\ \exists y : (y^2 = x) \vee (y^2 = xt)$$

True in $\mathbb{Z}$ [every integer is either even or odd]

but not in $\mathbb{Z}^2$.

Example

$$Th(\mathbb{Z}^m) = Th(\mathbb{Z}^n) \iff m = n$$

Question (Tarski): Suppose

$$Th(F_m) = Th(F_n)$$

how are m & n related?

Question: For which Γ,

$$Th(\Gamma) = Th(F_r) \text{ for some } r?$$

② Algebraic geometry over free groups.

.... trying to solve eqn's over free groups.

Solve systems of equations like:

$$\Phi \left\{ w_i : u_1 x_1 u_2 x_2 \dots u_r x_r = 1 \quad u_i \in F(a_i) \right.$$

Encode this as

$$G_\Phi = \langle a_1 \dots a_r, x_1 \dots x_s \rangle / w_j$$

Solving eqn's is the same as finding a homomorphism $G_\Phi \to F(a_i)$ w/ $a_i \to a_i$

$\Rightarrow$ wish to understand maps from a given group into free groups.

<u>Thm</u> (... Sela ...) $\forall$ f.g. Γ one can parametrize $Hom(\Gamma, F_r)$

Example $\mathrm{Th}(\pi_1(\Sigma_g)) = \mathrm{Th}(F_g) \quad g \geq 2.$