Why J?

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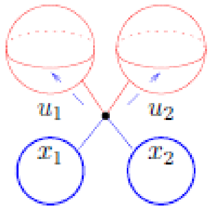
From the Hamburg handout:

Invariant #0. With Π_1 denoting "honest π_1", map $\gamma \in \mathcal{K}^{bh}(m,n)$ to the triple $(\Pi_1(\gamma^c), (u_i), (x_j))$, where the meridian of the balls u_i normally generate Π_1, and the "longtitudes" x_j are some elements of Π_1. $*$ acts like $*$, tm acts by "merging" two meridians/generators, hm acts by multiplying two longtitudes, and hta^{xu} acts by "conjugating a meridian by a longtitude":



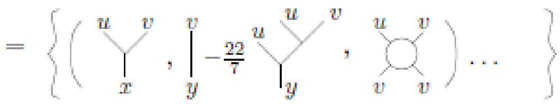
Not computable! (but nearly)

$$(\Pi, (u, \dots), (x, \dots)) \mapsto (\Pi * \langle \bar{u} \rangle / (u = x\bar{u}x^{-1}), (\bar{u}, \dots), (x, \dots))$$

Failure #0. Can we write the x's as free words in the u's? If $x = uv$, compute $x \mathbin{/\!/} hta^{xu}$:

$$x = uv \to \bar{u}v = u^{\bar{x}}v = u^{\bar{u}v}v = u^{u^{\bar{x}}v}v = u^{u^{u^{\bar{x}}v}v}v = \cdots$$

The Meta-Group-Action M. Let T be a set of "tail labels" ("balloon colours"), and H a set of "head labels" ("hoop colours"). Let $FL = FL(T)$ and $FA = FA(T)$ be the (completed graded) free Lie and free associative algebras on generators T and let $CW = CW(T)$ be the (completed graded) vector space of cyclic words on T, so there's $\mathrm{tr} : FA \to CW$. Let $M(T,H) := \{(\bar{\lambda} = (\lambda_x)_{x \in H}, \omega) : \lambda_x \in FL, \omega \in CW\}$



Operations. Set $(\bar{\lambda}_1, \omega_1) * (\bar{\lambda}_2, \omega_2) := (\bar{\lambda}_1, \bar{\lambda}_2, \omega_1 + \omega_2)$ and with $\mu = (\bar{\lambda}, \omega)$ define

$$tm_w^{uv} : \mu \mapsto \mu \mathbin{/\!/} (u, v \mapsto w),$$

$$hm_z^{xy} : \mu \mapsto \left(\left(\dots, \widehat{\lambda_x}, \widehat{\lambda_y}, \dots, \mathrm{bch}(\lambda_x, \lambda_y)_z \right), \omega \right)$$

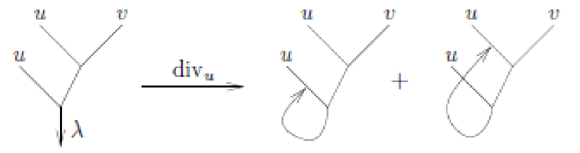
$$hta^{xu} : \mu \mapsto \underbrace{\mu \overbrace{\mathbin{/\!/\!/}}^{\text{"stable apply"}} (u \mapsto e^{\mathrm{ad}\,\lambda_x}(\bar{u})) \mathbin{/\!/} (\bar{u} \mapsto u)}_{\mu \mathbin{/\!/} CC_u^{\lambda_x}} + \underbrace{(0, J_u(\lambda_x))}_{\text{the "}J\text{-spice"}}$$

The Meta-Cocycle J. Set $J_u(\lambda) := J(1)$ where

$$J(0) = 0, \qquad \lambda_s = \lambda \mathbin{/\!/} CC_u^{s\lambda},$$

$$\frac{dJ(s)}{ds} = (J(s) \mathbin{/\!/} \mathrm{der}(u \mapsto [\lambda_s, u])) + \mathrm{div}_u \lambda_s,$$

and where $\mathrm{div}_u \lambda := \mathrm{tr}(u\sigma_u(\lambda))$, $\sigma_u(v) := \delta_{uv}$, $\sigma_u([\lambda_1, \lambda_2]) := \iota(\lambda_1)\sigma_u(\lambda_2) - \iota(\lambda_2)\sigma_u(\lambda_1)$ and ι is the inclusion $FL \hookrightarrow FA$:



Claim. $CC_u^{\mathrm{bch}(\lambda_1,\lambda_2)} = CC_u^{\lambda_1} \mathbin{/\!/} CC_u^{\lambda_2 /\!/ CC_u^{\lambda_1}}$ and

$$J_u(\mathrm{bch}(\lambda_1, \lambda_2)) = J_u(\lambda_1) \mathbin{/\!/} CC_u^{\lambda_2 /\!/ CC_u^{\lambda_1}} + J_u(\lambda_2 \mathbin{/\!/} CC_u^{\lambda_1}),$$

and hence tm, hm, and hta form a meta-group-action.

"Property M"

~~Perhaps free balloon?~~ not reasonable.

Is there a distinguished free loop in a Kbh?
(or a sum thereof?)

$$x = u^x v = x^{-1} u x v$$

$$x^2 = u x v$$

Is there a sense by which, in the presence of w all λ_x's are "finite"?

Question. What is dJ_{u}, the differential of J_{u_0}?

$$J_u : FL \longrightarrow CW \qquad dJ_u : FL \longrightarrow CW$$

The proof of property M should be in terms of meta-connections & meta-curvature.

Continued 2012-09