math.toronto.edu/drorbn/Talks/Hamburg-1208

What is Categorification = Concretization = de-abstraction?

"3" is {cow, cow, cow} and {pig, pig, pig} ...

Categorification is choosing which 3 it is!

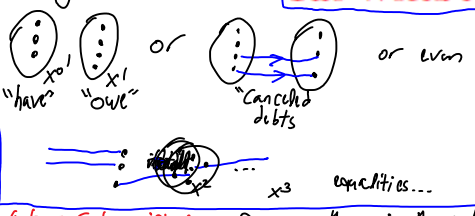
ℕ. Natural numbers ↦ Finite sets

equalities ⟶ bijections:

$\binom{2n}{n} = \sum \binom{n}{k}^2 \to \left(\begin{smallmatrix} X^{(1)} \cup X^{(2)} \\ |X| \end{smallmatrix}\right) \leftrightarrow \bigcup \binom{X}{k} \times \binom{X}{k}$

inequalities ⟶ injections & surjections

ℤ. Negative numbers:



The World According to Garp

Winter Categorification. Do something in the cat of vector spaces:

Khovanov: $K(L)$ is a chain complex of graded $\mathbb{Z}$-modules;

$V = \mathrm{span}\langle v_+, v_- \rangle$; $\deg v_\pm = \pm 1$; $q\dim V = q + q^{-1}$;

$K(\bigcirc^k) = V^{\otimes k}$; $K(\times) = \mathrm{Flatten}\left(0 \to \underset{\text{height } 0}{K(\,)\{1\}} \to \underset{\text{height } 1}{K(\asymp)\{2\}} \to 0\right)$;

$K(\times) = \mathrm{Flatten}\left(0 \to \underset{\text{height } -1}{K(\asymp)\{-2\}} \to \underset{\text{height } 0}{K(\,)\{-1\}} \to 0\right)$;

$\longrightarrow (V \otimes V \xrightarrow{m} V)$ $\quad m: \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0 \end{cases}$

$\longrightarrow (V \xrightarrow{\Delta} V \otimes V)$ $\quad \Delta: \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases}$

Example: $= q + q^3 + q^5 - q^9.$

What is categorification = concretization = de-abstraction?

"3" is {cow, cow, cow} and {pig, pig, pig} and ....

"Categorification": Answering "which 3 is it?"

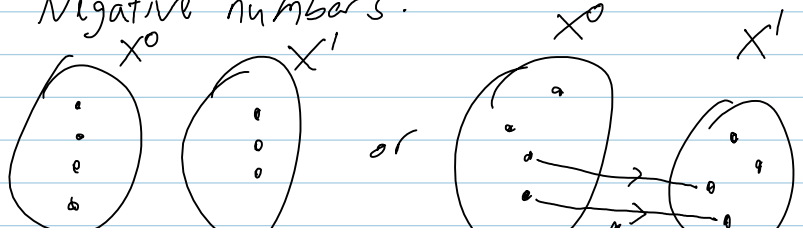
Natural numbers ⟶ finite sets

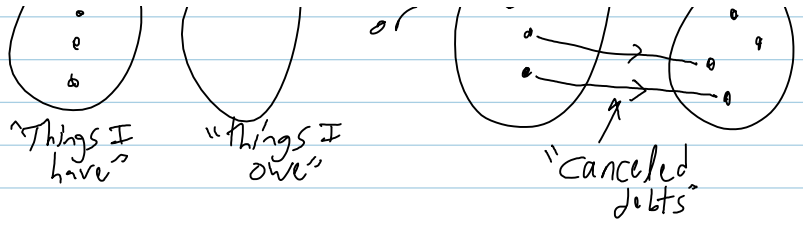
equalities ⟶ bijections:

$\binom{2n}{n} = \sum \binom{n}{k}^2 \qquad \left(\begin{smallmatrix} X^{(1)} \cup X^{(2)} \\ |X| \end{smallmatrix}\right) \leftrightarrow \bigcup \binom{X}{k} \times \binom{X}{k}$

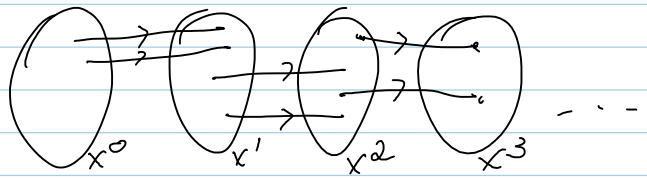
inequalities ⟶ injections & surjections.

Negative numbers:





or even



still should talk about "equalities"

Weaker categorification: Do the same in the category of vector spaces.

"3" $\longrightarrow$ V s.t. $\dim V = 3$, or better,

$\to V^{r-1} \xrightarrow{d} V^r \xrightarrow{d} V^{r+1} \to$ s.t. $d^2 = 0$

& $\chi(V^\bullet) := \sum (-1)^r \dim V^r = 3 = \sum (-1)^r \dim H^r$

equalities $\longrightarrow$ homotopies between complexes

Categorifying $\mathbb{Z}[q]$:

$f = \sum a_j q^j \longrightarrow V = \bigoplus V_j$ s.t. $q\dim V := \sum q^j \dim V_j = f$

or better

$\to V^{r-1} \xrightarrow{d} V^r \to V^{r+1} \to$ s.t. $d^2 = 0$ & $\deg d = 0$

& $\chi_q(V^\bullet) := \sum (-1)^r q\dim V^r = f = \sum (-1)^r q\dim H^r$

http://fiendx.wordpress.com/2010/07/15/the-world-according-to-garp/