Idea. "From BF to A"

[Explain why from BF we expect a morphism
out of the topological meta-bicrossed-products
of balloons & nooses, then state a
"conjecture" for what it is]

$$\mathcal{L} = \int B \wedge (dA + [A, A]) \qquad A \in \mathcal{R}^{1}$$
$$B \in \mathcal{R}^{n-2} \sim \mathcal{R}^{2}$$

"categorifiers take note"

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Idea. "Balloons & Hoops & their universal finite
type invariant"

Title. Balloons and Hoops and their Universal Finite -Type Invariant, BF Theory, and an Ultimate Alexander Invariant.

Abstract. Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4 - space behave much like the first and second fundamental groups of a topological space - hoops can be composed like in Π_1, balloons like in π_2, and hoops "act" on balloons as π_1 acts on π_2. We will observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.

We give an ansatz for a tree and wheel (that is, free -Lie and cyclic word) -valued invariant Z of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that Z is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of Z is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable , if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least -wasteful in a computational sense. If you believe in categorification, here's a wonderful playground.

1. Explain the knotting of hoops and balloons
in $\mathbb{R}^4$ and their similarity with Π_1
& Π_2

2. Explain how tangles map into the above; likewise for "weakly virtual" tangles. Ask a topology question...

3. Construct by meta-means the universal finite type invariant... To do ✓

Ask: "This is the complete evaluation of something. Is it BF theory~?

4. State the relationship with arrow diagrams and mention that they look like Feynman diagrams for $B \wedge F_A$.

5. Write the formulas for the reduction mod To do again $[x,y] = c_y x - c_x y$, mention Alexander & polynomiallity & "efficiency". Ask: Can you categorify this?

6. Ask can you generalize this to v-knots? Get similar formulas for Jones? Relate this with Etingof-Kazhdan?