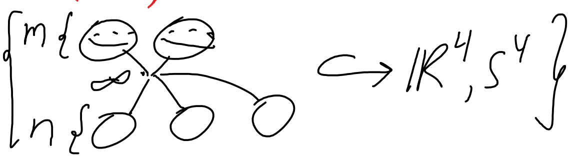
Balloons and Hoops and their Universal Finite–Type Invariant, BF Theory, and an Ultimate Alexander Invariant

B&H

Dror Bar–Natan in Hamburg, August 2012
http://www.math.toronto.edu/~drorbn/Talks/Hamburg-1208/

Abstract. Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space - hoops can be composed like in π_1, balloons like in π_2, and hoops "act" on balloons as π_1 acts on π_2. We will observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.
We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word) -valued invariant Z of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that Z is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of Z is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, here's a wonderful playground.

$K^{bh}(m,n)$.



Example. use band notation

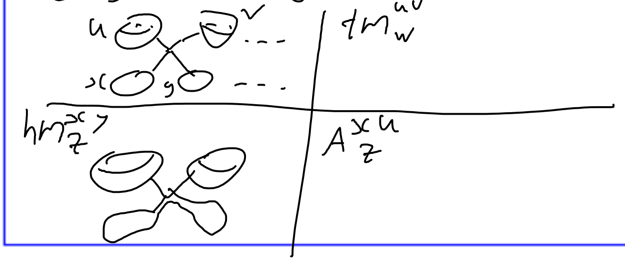
$\to$ knotted tubes $\to$ K^{bh}

* u-tangles inject into K^{bh}
* in fact, n-comp v/w tangles $\to K^{bh}(n,n)$

we have a conjectural understanding of a.

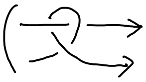
Tcirc

Meta-group-action If X is a space, $\pi_1(X)$ is a group, $\pi_2(X)$ is an Abelian group, and π_1 acts on π_2



The meta-group-action $A^{bh}(m,n)$

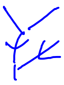
The invariant ζ

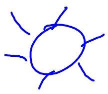
Example: $\zeta($  $)$

(The log of)

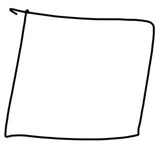
Claim. ζ is a "universal finite type invariant"

It would be nice to have a precise statement of this conjecture.

$Lie_n =$ 

$tr_n =$ 

ζ and BF



Cattaneo

} modulo the Alexander relation

}/A, repackaged

Polynomiality, efficiency, categorification

Gukov Morrison

Further directions.

v-knots?

Jones?

AT & KV?

E-K?

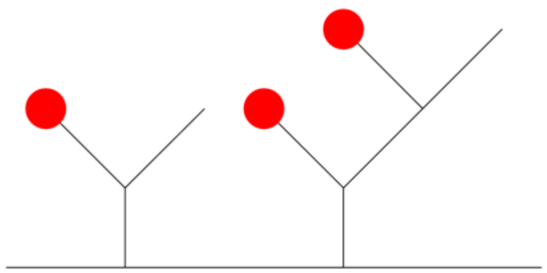
"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

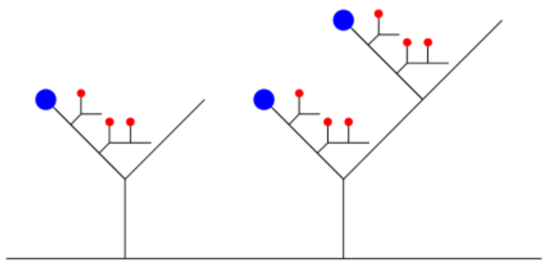
must put pictures of A&T somewhere.

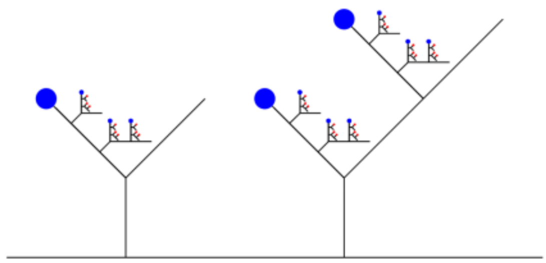
Somewhere:

Proof. 1. I can prove.

2. Who needs a proof when you can compute?







Two further pictures like this are needed: * proof of the BCH property.

* def of j.

no