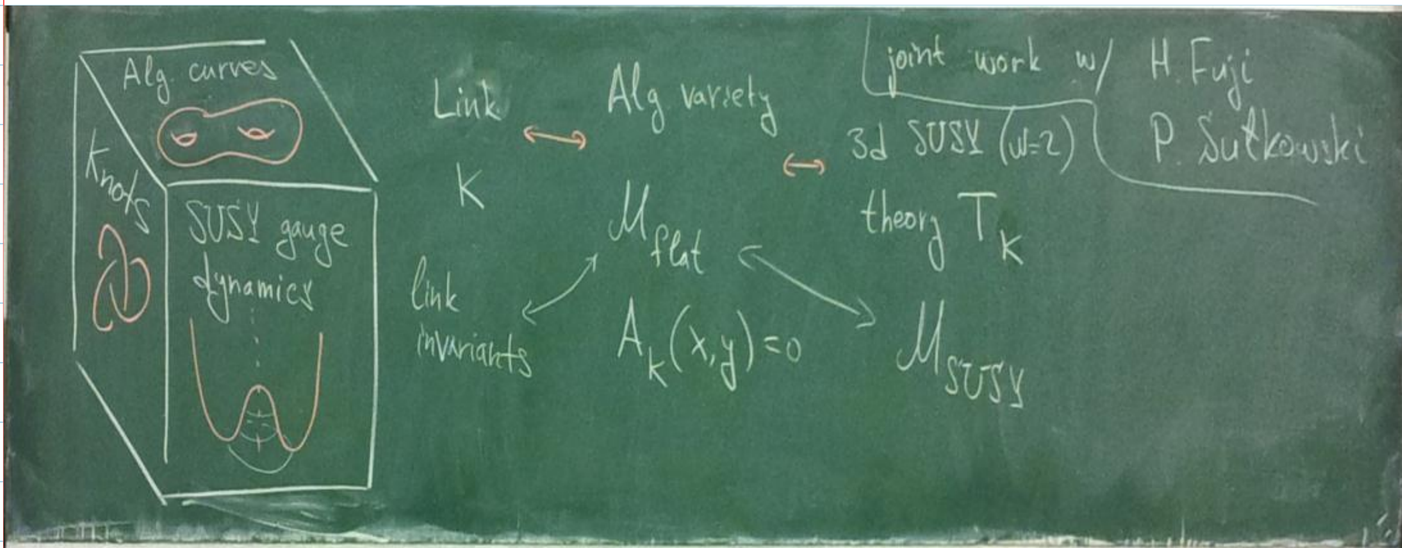
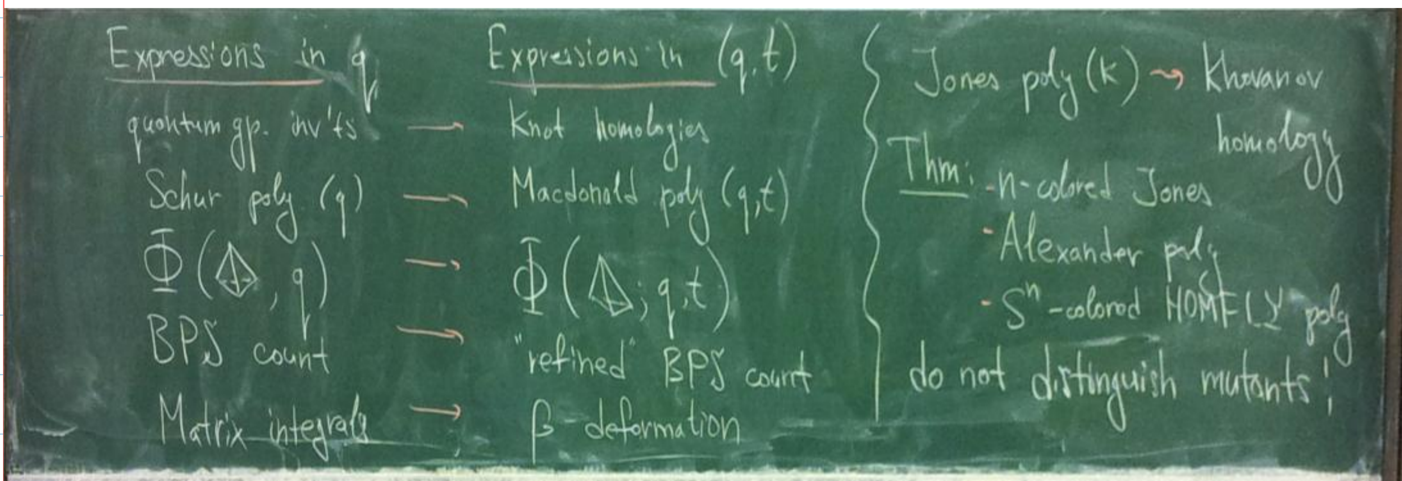
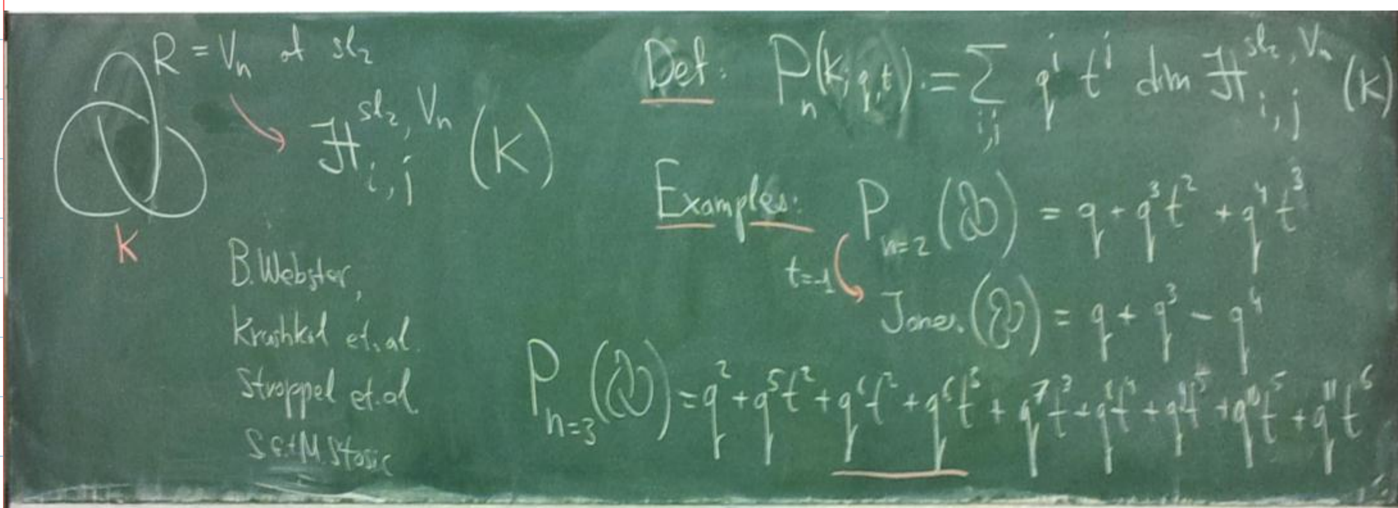
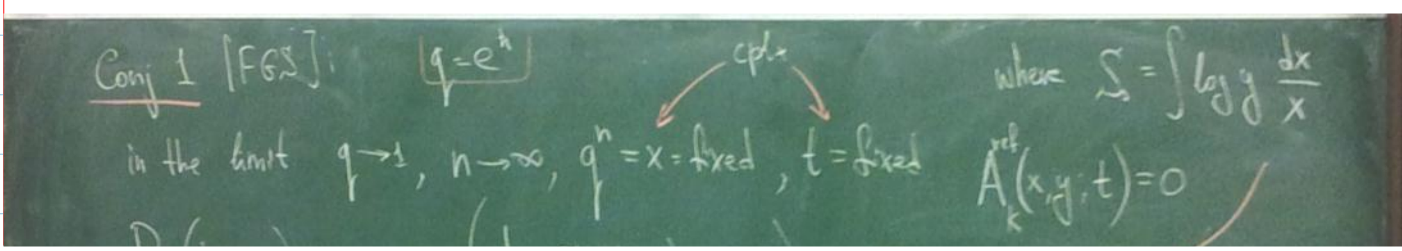
Gukov's Talk

August-30-12
5:02 PM









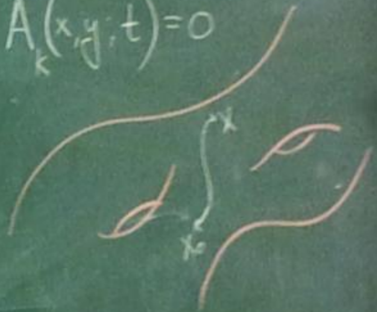
Conj 1 [FGS]: $q = e^{\hbar}$

in the limit $q \to 1$, $n \to \infty$, $q^n = x$ = fixed, t = fixed (cplx)

$$P_n(K;q,t) \simeq \exp\left(\frac{1}{\hbar} S_0(x;t) + \dots\right)$$

where $S = \int \log y \frac{dx}{x}$

$A_K^{ref}(x,y;t) = 0$



Conj. 2 [FGS]: $a_k P_{n+k}(q,t) + \dots + a_1 P_{n+1}(q,t) + a_0 P_n(q,t) = 0$

$\hat{A}(x,y;q,t) \xrightarrow{q\to 1} A^{ref}(x,y;t)$

$a_i(x,q,t)$ = rational functions of $q^n = x$, q, t

⇕

$$\hat{A}(x,y;q,t) = \sum_{i=0}^{k} a_i(x;q,t)\, \hat{y}^k \qquad \hat{y}\hat{x} = q\hat{x}\hat{y}$$

$$\hat{A} P_* = 0$$

$A^{super}(x,y,t,a)$

$a=1$ → $A^{ref}(x,y;t)$ $\qquad$ $t=-1$ → $A^{Q\text{-def}}(x,y,a)$

$t=-1$ → $A_K(x,y)$ ← $a=1$

Ex: $A(\theta) = (y-1)(y+x^3)$

$$A^{super}(x,y;a,t) = (1+at^3x)\,y^2 - a\left(1 - t^2x + 2t^2(1+at)x^2 + at^5x^3 - a^2t^5x^4\right)y + a^2t^4(x-1)x^3$$

$A^{super}(x,y;a,t)$

⇝

$\hat{A}^{super}(\hat{x},\hat{y};a,t,q)$ (a, t: comm.; q: non-comm.)

$\hat{y}\hat{x} = q\hat{x}\hat{y}$

$a_k P_{n+k}(a,q,t) + \dots + a_1 P_{n+1}(a,q,t) + a_0 P_n(a,q,t) = 0$

⇕

$\hat{A}^{super} P_n = 0$

Eynard-Orantin

S.G. P. Sułkowski

Borot-Eynard

solve this rec for $P_{n<0}(a,q,t) = 0$

$P_2 = aq^{-1} + aqt^2 + a^2t^3 \qquad P_1(a,q,t) = 1$

$$P_3 = a^2q^{-2} + a^2q(1+q)t^2 + a^3(1+q)t^3 + a^2q^4t^4 + a^3q^3(1+q)t^5 + a^4q^3t^6$$