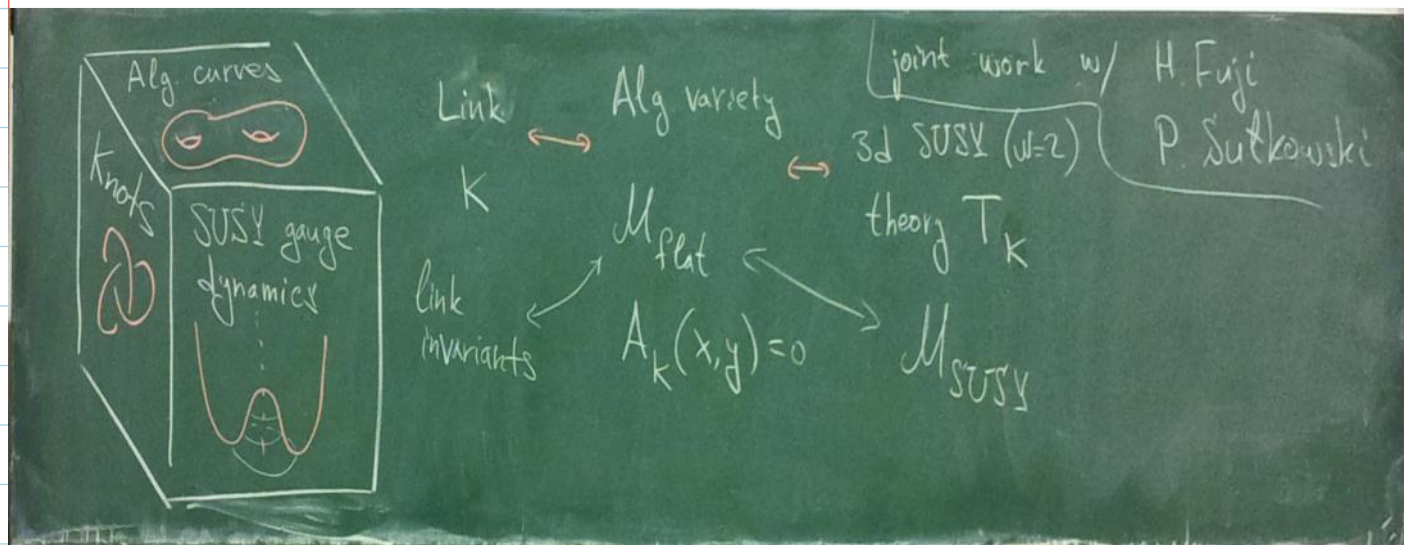


Gukov's Talk

August-30-12
5:02 PM



Expressions in q	Expressions in (q,t)	Notes
quantum gp. inv'ts	Knot homologies	Jones poly (K) → Khovanov homology Thm: n -colored Jones - Alexander poly. - S^n -colored HOMFLY poly. do not distinguish mutants!
Schur poly (q)	Macdonald poly (q,t)	
$\Phi(\Delta, q)$	$\Phi(\Delta; q,t)$	
BPS count	"refined" BPS count	
Matrix integrals	β -deformation	

$R = V_n$ at sl_2 → $\mathcal{H}_{i,i}^{sl_2, V_n}(K)$

Def: $P_n(K; q,t) = \sum_{i,j} q^i t^j \dim \mathcal{H}_{i,j}^{sl_2, V_n}(K)$

Examples:

- $P_{n=2}(\mathcal{D}) = q + q^3 t^2 + q^4 t^3$
- $t=-1$ → Jones (\mathcal{D}) = $q + q^3 - q^4$
- $P_{n=3}(\mathcal{D}) = q^2 + q^5 t^2 + q^6 t^2 + q^6 t^5 + q^7 t^3 + q^8 t^3 + q^8 t^5 + q^8 t^5 + q^8 t^5$


B. Webster, Kravtsov et al., Stroppel et al., S & M. Stasheff

Conj 1 [FGS]: $q = e^h$
 in the limit $q \rightarrow 1, n \rightarrow \infty, q^n = x = \text{fixed}, t = \text{fixed}$ where $\Sigma = \int \log q \frac{dx}{x}$
 $A_K^{ref}(x,y,t) = 0$

Conj 1 [FGS]: $q = e^h$ cplx where $\zeta = \int \log y \frac{dx}{x}$

in the limit $q \rightarrow 1, n \rightarrow \infty, q^n = x = \text{fixed}, t = \text{fixed}$ $A_k^{\text{ref}}(x, y; t) = 0$

$P_n(k; q, t) \approx \exp\left(\frac{1}{h} \zeta_0(x; t) + \dots\right)$



Conj 2 [FGS]: $a_k P_{n+k}(q, t) + \dots + a_2 P_{n+1}(q, t) + a_0 P_n(q, t) = 0$

$\hat{A}(x, y; q, t) \xrightarrow{q \rightarrow 1} A(x, y; t)$ $a_i(x, q, t) = \text{rational functions of } q^n = x, q, t$

$\hat{A}(x, y; q, t) = \sum_{i=0}^k a_i(x, q, t) \hat{y}^i$ $\hat{y} \hat{x} = q \hat{x} \hat{y}$

$\hat{A} P_x = 0$

Ex: $A(\emptyset) = (y-1)(y+x^3)$

$A^{\text{super}}(x, y; a, t) = (1+at^3x)y^2 - a(1-t^2x+2t^2(1+at)x^2+at^5x^3-a^2t^5x^4)y + a^2t^4(x-1)x^3$

Diagram showing relationships between $A^{\text{super}}(x, y; a, t)$, $A^{\text{ref}}(x, y; t)$, $A^{\text{odd}}(x, y; a)$, and $A(x, y)$ with arrows labeled $a=1$ and $t=1$.

$A^{\text{super}}(x, y; a, t)$

$\hat{A}^{\text{super}}(\hat{x}, \hat{y}; a, t, q)$ comm.

$\hat{y} \hat{x} = q \hat{x} \hat{y}$ non-comm.

$a_k P_{n+k}(a, q, t) + \dots + a_2 P_{n+1}(a, q, t) + a_0 P_n(a, q, t) = 0$

$\hat{A}^{\text{super}} P_n = 0$

Eynard-Orantin
S.G. P. Subkowitzki
Borot-Eynard

solve this rec for $P_{n < 0}(a, q, t) = 0$

$P_2 = aq^{-1} + aqt^2 + a^2t^3$ $P_1(a, q, t) = 1$

$P_3 = a^2q^{-2} + a^2q(1+q)t^2 + a^3(1+q)t^3 + a^2q^{-1}t^4 + a^3q^3(1+q)t^5 + a^4q^3t^6$