Andersen: A Geometric Construction of the Reshetikhin-Turaev TQFT

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3:59 AM

Witten '88: Chern-Simons Theory in 3D, $G = SU(2)$

R.T.: Combinatorial construction with

$$U_q(Sl(2,\mathbb{C})) \quad q = e^{2\pi i/k+2}$$

Geometric Construction: Geometric quantization of a certain moduli space: W, Axelrod, Della-Pietro, Hitchin.

Σ: closed oriented surface.

M = Moduli space of flat $SU(2)$-connections

$$= \mathrm{Hom}(\pi_1(\Sigma), SU(2))/SU(2)$$

$$= \{(A_i, B_i)_{i=1}^{g} \in SU(2)^{2g} : \prod [A_i, B_i] = 1\}/SU(2)$$

$\dim M = 6g - g$

$M' \subseteq M$ The irreducible reps.

Geometric Quantization: This is a symplectic manifold, w/ the Goldman structure ω, s.t.

$$[\omega] \in H^2(M, \mathbb{Z})$$

There is a Hermitian line bundle

$$\begin{array}{ccc} \mathbb{C} & \longrightarrow & \mathcal{L} \\ & & \downarrow \\ & & M \end{array}$$

∇ : connection

$\langle \cdot , \cdot \rangle$ Hermitian structure.

$F_\nabla = -i\omega$

Γ = mapping class group of Σ.

Γ acts on M & $\exists$ lift of Γ's action to $(\mathcal{L}, \nabla, \langle \cdot , \cdot \rangle)$

$$\mathcal{H}^{(k)} := C^\infty(M, \mathcal{L}^{\otimes k})$$

Kähler quantization: Find a complex structure I invariant under Γ, restrict to holomorphic sections.

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