Ktgs - referees' comments July-09-12 5:32 PM Ref.: Ms. No. JKTR-D-11-00094 Homomorphic expansions for knotted trivalent graphs Journal of Knot Theory and Its Ramifications Dear Ms. Zsuzsanna Dancso, Reviewers have now submitted their comments on your paper. You will see that they are advising you of a minor revision. For your guidance, their comments are appended below. Please submit a list of changes or a rebuttal against each point raised when you submit the final version of the manuscript. Your revision is due by Aug 16, 2012. If you need more time for the revision, please do not hesitate to contact me. To submit a revision, go to http://jktr.edmgr.com/ and log in as an Author. You will see a menu item "Submission Needing Revision". You will find your submission record there. Yours sincerely J. Scott Carter, Ph.D. Managing Editor Journal of Knot Theory and Its Ramifications Reviewers' comments: Reviewer #1: In this paper, the authors generalize the universal finite type invariant \$Z^{old}\$ of Knotted Trivalent Graphs (KTG) so that the resulting invariant is "very well-behaved". First, generalize KGT to dotted KGT (dKGT) and extend \$Z^{old}\$ by substituting the element

\$\nu^{\pm1/2}\$ to the dots where \$\nu\$ is \$Z^{old}\$ of the trivial knot.

In my opinion, the idea given in this paper is a good method to handle \$Z^{old}\$ rather than a generalization of the invariant. The first part of this paper is very lengthy. However, the contents of the latter half show various useful properties of \$Z^{old}\$.

Reviewer #2: \documentclass{report}

\usepackage{amssymb}
\setlength{\voffset}{-1cm}
\setlength{\hoffset}{-1.5cm}
\addtolength{\textwidth}{3cm}
\addtolength{\textheight}{2cm} \setlength{\parindent}{0pt}
\pagestyle{empty}
\begin{document}
\section*{Comments on the paper ``Homomorphic expansions for knotted
trivalent graphs}''}
\vspace{0.5cm}
(repace (e.e.e.m)
\section*{Questions, remarks, suggestions}
Abstract \& Introduction: A reference to the work of ChepteaLe (Comm.
Math. Phys. 2007) could be added.\\
P.2, I3: ``a cyclic ordering of the three edges'' \$\leadsto\$ ``a cyclic
ordering of the three half-edges'' (since, technically, there may be some looped edges.)\\
(since, technically, there may be some looped edges.) \\
P.3, I13: Although it is clear from the context, I can't see where the
notation \$\mathcal{K}(\Gamma)\$ has been set.\\
P.10, I13: The sentence ``(This is because \dots opposite
orientation.)" could be moved to 1.14,
where the fact that \$S(\nu)=\nu\$ is already used.\\
P.12, I1 \& I2: Should you not here refer to Lemma 3.2 instead of
Theorem 3.3? As for the dotted edge connected sum,
you could also say that it is a special case of tree connected sum, which is an operation of dKTG.\\
is an operation of dk1d.\\
P.13, proof of Proposition 3.5: Make explicit the algebra
\$\mathcal{A}(\Gamma)\$ where the identity \$\Psi^2=\Psi\$ holds.\\
P.14, I18: Although it is clear from the context, I can't see where the
notation \$\mathcal{A}_m\$ has been set.\\
P.15, Theorem 4.1: What is the relation between the associator \$\Phi\$
. 125, Theorem 112. What is the relation between the associator 4 (1 mg

produced by \$Z\$ and the associator
from which the construction of \$Z^{old}\$ starts in [MO]? Is Theorem 4.1
a way to make associators more symmetric?\\
P.1619, proof of Theorem 4.1: The maximal trees in the depicted KTGs are
not always easy to distinguish
from the rest of the graph. It could be helpful to draw them in a thicker
way.\\
P.19, bottom figure: The crosses are missing on the tetrahedra.\\
1.13, bottom figure. The crosses are missing on the tetraheara. (
P.20, middle figure: The indication of the vertex orientations is missing
on the theta graph with twisted edges.\\
P.21, proof of Theorem 2.1: It seems that the inclusion \$\mathcal{I}
\subset \mathcal{F}_1\$ is not correct
because your objects are framed.
For instance, the difference \$U_0-U_1\$ (where \$U_k\$ denotes the \$k\$-framed unknot) belongs to
\$\mathcal{I}\$ but does not belong to \$\mathcal{F}_1\$.
I would say that, in your definition of the Vassiliev filtration
\$\mathcal{F}\$,
you need to include the framing change move.\\
P.23, I.7: I don't see why \$\nu:=Z(\circlearrowleft)\$ ``is by definition
an invertible element" of \$\mathcal{A}(\circlearrowleft)\$
(\dots although I agree that one expects \$Z\$ to have group-like values).
\newpage
(Hewpage
\section*{Typos}
P.3, I.2: ``obtained from it by ``thickening vertices'' \$\leadsto\$
``obtained from it by ``thickening'' vertices''\\
P.4, l.14: "by resolutions of of \$n\$-singular immersions" \$\leadsto\$
``by resolutions of \$n\$-singular immersions''\\
P.4, I16: ``A chord diagram'' \$\leadsto\$ ``A \emph{chord diagram}'' (
this is a definition)\\
, , ,
P.5, I18: ``\$\mathcal{A}(O)
' \$\leadsto\$ ``\$\mathcal{A}(\circlearrowleft)
(KTGs, and in particular, knots are oriented in your paper)\\

```
P.8, I.11: \c_{d,a}: \mathcal{K}(\Gamma) \to \mathcal{K}(c_{v,a} \Gamma)
$\leadsto$
``$c {d,a}: \mathcal{K}(\Gamma) \to \mathcal{K}(c {d,a} \Gamma)
'\\
P.8, footnote: ``the KTGs $\gamma_1$ and $\gamma_2$ each embedded''
$\leadsto$
"the KTGs $\gamma 1$ and $\gamma 2$ are each embedded"\\
P.13, footnote: Some quotation marks are missing at the end of the
sentence.\\
P.14, I.7: ``involving $\Phi \in \mathcal{A}(\uparrow 3)$ and $R\in
\mathcal{A}(\uparrow_2)
$\leadsto$
 "involving $\Phi \in \mathcal{A}(\uparrow 3)$ and an extra $R\in
\mathcal{A}(\uparrow_2)
(... in constrast with $\Phi$, $R$ has not been discussed yet)\\
P.15, I.3: ``made only of only horizontal chords'' $\leadsto$ ``made only
of horizontal chords"\\
P.16, middle figure: third arrow ``$\stackrel{\dot u}{\longrightarrow}
$\leadsto$ ``$\stackrel{c^2}{\longrightarrow}
'\\
P.17, middle figure: third arrow ``$\stackrel{\dot u}{\longrightarrow}
$\leadsto$ ``$\stackrel{c^2}{\longrightarrow}
//
P.18, top figure: first arrow ``$\stackrel{c^5 \circ
\sharp^2}{\longrightarrow}
$\leadsto$ ``$\stackrel{c^4 \circ \sharp^2}{\longrightarrow}
'\\
P.18, top figure: second arrow ``$\stackrel{\dot u^2}{\longrightarrow}
$\leadsto$ ``$\stackrel{c^4 \circ \dot u^2}{\longrightarrow}
H'
P.19, I.-14: ``The $Z$ value graph'' $\leadsto$ ``The $Z$ value of the
```

