Pensieve header: Perturbative β-calculations.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.0"];
<< betaCalculus.m
```

```
Clear[ħ];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
   βSimplify[ω],
   βSimplify[μ]
  ];
```

The Knot-Theoretic Equations

```
{
  V0 = βCollect[
     B[ω[ħ c1, ħ c2], α[ħ c1, ħ c2] t[1] h[1] +
       β[ħ c1, ħ c2] t[1] h[2] + γ[ħ c1, ħ c2] t[2] h[1] + δ[ħ c1, ħ c2] t[2] h[2]]
    ] /. {
     (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0,
     (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
    },
  C0 = βCollect[B[κ[ħ c1], 0]] /. {
     (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0,
     (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
    },
  eqns1 = HardR4[V0],
  eqns2 = TwistEq[V0],
  eqns3 = And[(V0 // dη[1]) == B[1, 0], (V0 // dη[2]) == B[1, 0]],
  eqns4 = V0 ** (V0 // dA[1] // dA[2]) == B[1, 0],
  eqns5 = CapEquation[V0, C0],
  eqns6 = (C0 // tη[1]) == B[1, 0],
  eqns7 = (V0 == Rot120[V0]),
  eqns8 = V0 ** (V0 // dS[1] // dS[2]) == R[1, 2]
 } // ColumnForm
```

A very large output was generated. Here is a sample of it:

<<1>>

Show Less | Show More | Show Full Output | Set Size Limit...

```
eqns = (eqns1 && eqns2 && eqns3 && eqns4 && eqns5 && eqns6 && eqns8);

Timing[sol = PerturbativeSolveAlways[eqns, ħ, $PerturbativeDegree, {c1, c2}]]
```

$$\Big\{4.867,\ \Big\{\alpha_0\to 0,\ \beta_0\to \frac{1}{2},\ \gamma_0\to 0,\ \delta_0\to 0,\ \kappa_0\to 1,\ \omega_0\to 1,\ \alpha_{10}\to 0,$$

$$\beta_{10}\to \frac{1}{8}+\gamma_1,\ \beta_1\to \frac{1}{8}+\frac{1}{24}\,(-1+24\,\gamma_1),\ \delta_1\to 0,\ \alpha_1\to \frac{1}{8}+\delta_{10},\ \omega_1\to 0,\ \omega_{10}\to 0,$$

$$\kappa_2\to \frac{1}{16}\left(1+16\,\delta_{10}+16\,\kappa_1^2\right),\ \alpha_{20}\to 0,\ \delta_2\to 0,\ \omega_{11}\to \frac{1}{16}\,(1+16\,\delta_{10}),\ \alpha_2\to 0,\ \alpha_{11}\to 0,$$

$$\beta_{20}\to \frac{1}{24}\,(1+24\,\gamma_1),\ \beta_2\to \gamma_1,\ \gamma_2\to 0,\ \beta_{11}\to \frac{1}{48}\,(1+48\,\gamma_1),\ \gamma_{11}\to 0,\ \gamma_{20}\to 0,\ \delta_{11}\to -\frac{\gamma_1}{2},$$

$$\delta_{20}\to \frac{1}{24}\,(1-24\,\gamma_1+24\,\delta_{10}),\ \omega_2\to 0,\ \omega_{20}\to 0,\ \gamma_{10}\to \frac{1}{24}\,(-1+24\,\gamma_1),\ \alpha_{30}\to 0,$$

$$\beta_{30}\to \frac{1}{64}\left(1+48\,\gamma_1+\frac{1}{15}\,(-7+240\,\gamma_1+960\,\gamma_{30})\right),\ \beta_{21}\to \frac{1}{192}\,(1+144\,\gamma_1+192\,\gamma_{12}),$$

$$\beta_{12}\to \frac{1}{192}\left(-1+144\,\gamma_1+\frac{1}{5}\,(1-80\,\gamma_1+960\,\gamma_{12})\right),\ \beta_3\to \frac{1}{64}\,(-1+48\,\gamma_1+64\,\gamma_{30}),$$

$$\delta_3\to 0,\ \alpha_{21}\to \frac{1}{192}\,(-1+48\,\gamma_1+192\,\delta_{12}),\ \alpha_{12}\to \frac{1}{192}\left(-1+144\,\gamma_1+\right.$$

$$\left.\frac{1}{12}\left(-7-240\,\gamma_1-1152\,\gamma_1^2-48\,\delta_{10}+1152\,\gamma_1\,\delta_{10}+2304\,\delta_{10}^2+1728\,\delta_{12}+576\,\delta_{30}\right)\right),$$

$$\alpha_3\to \frac{1}{64}\,(-3+96\,\gamma_1-48\,\delta_{10}+64\,\delta_{30}),\ \kappa_3\to \frac{1}{16}\,\kappa_1\left(3+48\,\delta_{10}+16\,\kappa_1^2\right),\ \omega_3\to 0,$$

$$\omega_{12}\to 0,\ \omega_{21}\to 0,\ \omega_{30}\to 0,\ \alpha_{40}\to 0,\ \beta_{40}\to \frac{1}{120}\,(-1+120\,\gamma_1+240\,\gamma_{30}),\ \gamma_4\to 0,\ \delta_4\to 0,$$

$$\omega_{22}\to \frac{1}{64}\left(-1+40\,\gamma_1+192\,\gamma_1^2-8\,\delta_{10}-192\,\gamma_1\,\delta_{10}+128\,\delta_{10}^2+96\,\delta_{12}+32\,\delta_{30}\right),\ \alpha_4\to 0,\ \alpha_{13}\to 0,$$

$$\alpha_{22}\to 0,\ \alpha_{31}\to 0,\ \beta_4\to \frac{1}{480}\,(-7+240\,\gamma_1+960\,\gamma_{30}),\ \beta_{31}\to \frac{1}{384}\,(-1+240\,\gamma_1+576\,\gamma_{12}+192\,\gamma_{30}),$$

$$\gamma_{13}\to 0,\ \beta_{22}\to \frac{1}{960}\,(-3+400\,\gamma_1+1920\,\gamma_{12}),\ \gamma_{22}\to 0,\ \beta_{13}\to \frac{1}{128}\,(-1+48\,\gamma_1+192\,\gamma_{12}+64\,\gamma_{30}),$$

$$\gamma_{31}\to 0,\ \gamma_{40}\to 0,\ \delta_{13}\to \frac{7-240\,\gamma_1-960\,\gamma_{30}}{1920},\ \delta_{22}\to -\gamma_{12}+\delta_{12},\ \delta_{31}\to \frac{1}{7680}$$

$$\left(-47+1680\,\gamma_1-5760\,\gamma_1^2-11\,520\,\gamma_{12}-240\,\delta_{10}+5760\,\gamma_1\,\delta_{10}+11\,520\,\delta_{10}^2+8640\,\delta_{12}+2880\,\delta_{30}\right),$$

$$\delta_{40}\to \frac{1}{24}\,(-1+24\,\gamma_1-48\,\gamma_{30}-24\,\delta_{10}+48\,\delta_{30}),\ \omega_4\to 0,\ \kappa_4\to \frac{1}{256}$$

$$\left(-3+160\,\gamma_1+768\,\gamma_1^2-768\,\gamma_1\,\delta_{10}+768\,\delta_{10}^2+384\,\delta_{12}+128\,\delta_{30}+96\,\kappa_1^2+1536\,\delta_{10}\,\kappa_1^2+256\,\kappa_1^4\right),$$

$$\omega_{13}\to \frac{1}{128}\left(-3+80\,\gamma_1+384\,\gamma_1^2-48\,\delta_{10}-384\,\gamma_1\,\delta_{10}+192\,\delta_{12}+64\,\delta_{30}\right),$$

$$\omega_{31}\to \frac{1}{128}\left(-3+80\,\gamma_1+384\,\gamma_1^2-48\,\delta_{10}-384\,\gamma_1\,\delta_{10}+192\,\delta_{12}+64\,\delta_{30}\right),\ \omega_{40}\to 0,$$

$$\delta_{21}\to \frac{1}{2304}\left(-7-240\,\gamma_1-1152\,\gamma_1^2-48\,\delta_{10}+1152\,\gamma_1\,\delta_{10}+2304\,\delta_{10}^2+1728\,\delta_{12}+576\,\delta_{30}\right),$$

$$\gamma_{21}\to \frac{1}{960}\,(1-80\,\gamma_1+960\,\gamma_{12}),\ \gamma_3\to \frac{1}{960}\,(-7+240\,\gamma_1+960\,\gamma_{30}),$$

$$\beta_{50}\to\frac{1}{64}\left(-1+60\,\gamma_1+160\,\gamma_{30}+\frac{1}{21}\,(-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50})\right),$$

$$\beta_{14}\to\frac{1}{160}\,(-1+30\,\gamma_1+240\,\gamma_{12}+160\,\gamma_{30}+160\,\gamma_{41}),$$

$$\beta_5\to\frac{1}{96}\,(-1+30\,\gamma_1+240\,\gamma_{30}+96\,\gamma_{50}),\ \alpha_{41}\to\frac{1}{480}\left(-1+30\,\gamma_1+240\,\gamma_{30}+\right.$$
$$\left.\frac{1}{120}\left(-283+936\,\gamma_1-34\,560\,\gamma_1^2-691\,200\,\gamma_1^3+57\,600\,\gamma_{12}+552\,960\,\gamma_1\,\gamma_{12}-28\,800\,\gamma_{30}-\right.\right.$$
$$276\,480\,\gamma_1\,\gamma_{30}-7440\,\delta_{10}-172\,800\,\gamma_1\,\delta_{10}-414\,720\,\gamma_1^2\,\delta_{10}-276\,480\,\gamma_{12}\,\delta_{10}+138\,240$$
$$\gamma_{30}\,\delta_{10}+34\,560\,\delta_{10}^2+276\,480\,\gamma_1\,\delta_{10}^2-48\,960\,\delta_{12}-483\,840\,\gamma_1\,\delta_{12}-276\,480\,\delta_{10}\,\delta_{12}+$$
$$\left.\left.46\,080\,\delta_{23}+12\,480\,\delta_{30}+69\,120\,\gamma_1\,\delta_{30}-92\,160\,\delta_{10}\,\delta_{30}+23\,040\,\delta_{41}-11\,520\,\delta_{50}\right)\right),$$

$$\alpha_{32}\to\frac{1}{320}\,(-1+30\,\gamma_1+240\,\gamma_{12}+160\,\gamma_{30}+320\,\delta_{23}),\ \alpha_{23}\to$$
$$\frac{1}{3840}\left(1+8640\,\gamma_{12}-2880\,\delta_{12}+\frac{2}{3}\left(3+17\,\gamma_1+1680\,\gamma_1^2+17\,280\,\gamma_1^3-2400\,\gamma_{12}-23\,040\,\gamma_1\,\gamma_{12}+\right.\right.$$
$$1600\,\gamma_{30}+15\,360\,\gamma_1\,\gamma_{30}+231\,\delta_{10}+3840\,\gamma_1\,\delta_{10}+11\,520\,\gamma_1^2\,\delta_{10}+11\,520\,\gamma_{12}\,\delta_{10}-$$
$$7680\,\gamma_{30}\,\delta_{10}-720\,\delta_{10}^2+5760\,\gamma_1\,\delta_{10}^2+11\,520\,\delta_{10}^3+2400\,\delta_{12}+20\,160\,\gamma_1\,\delta_{12}+$$
$$\left.\left.2880\,\delta_{10}\,\delta_{12}+3840\,\delta_{23}-640\,\delta_{30}-4800\,\gamma_1\,\delta_{30}+960\,\delta_{10}\,\delta_{30}+1920\,\delta_{41}\right)\right),$$

$$\alpha_{14}\to\frac{1}{2560}\left(9-560\,\gamma_1+1920\,\gamma_1^2+7680\,\gamma_{12}+1280\,\gamma_{30}+80\,\delta_{10}-1920\,\gamma_1\,\delta_{10}-\right.$$
$$\left.3840\,\delta_{10}^2-2880\,\delta_{12}-960\,\delta_{30}+2560\,\delta_{41}\right),$$

$$\alpha_5\to\frac{1}{128}\,(7-160\,\gamma_1+640\,\gamma_{30}+200\,\delta_{10}-320\,\delta_{30}+128\,\delta_{50}),\ \alpha_{50}\to 0,$$

$$\beta_{41}\to\frac{1}{960}\left(-7+540\,\gamma_1+1440\,\gamma_{12}+960\,\gamma_{30}+\frac{1}{7}\,(3-196\,\gamma_1+3360\,\gamma_{12}+6720\,\gamma_{41})\right),$$

$$\beta_{32}\to\frac{1}{1920}\,(-7+600\,\gamma_1+4320\,\gamma_{12}+1920\,\gamma_{23}+480\,\gamma_{30}),\ \beta_{23}\to$$
$$\frac{1}{1920}\left(-7+360\,\gamma_1+4320\,\gamma_{12}+480\,\gamma_{30}+\frac{2}{21}\,(-13+420\,\gamma_1-5040\,\gamma_{12}+20\,160\,\gamma_{23}+1680\,\gamma_{30})\right),$$

$$\delta_5\to 0,\ \omega_5\to 0,\ \omega_{14}\to 0,\ \omega_{23}\to 0,\ \omega_{32}\to 0,\ \omega_{41}\to 0,\ \omega_{50}\to 0,$$

$$\kappa_5\to\frac{1}{256}\,\kappa_1\left(-15+800\,\gamma_1+3840\,\gamma_1^2-3840\,\gamma_1\,\delta_{10}+3840\,\delta_{10}^2+\right.$$
$$\left.1920\,\delta_{12}+640\,\delta_{30}+160\,\kappa_1^2+2560\,\delta_{10}\,\kappa_1^2+256\,\kappa_1^4\right),\ \alpha_{60}\to 0,\ \delta_6\to 0,$$

$$\omega_{33}\to\frac{1}{7680}\left(49-748\,\gamma_1-17\,760\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+\right.$$
$$61\,440\,\gamma_1\,\gamma_{30}+1380\,\delta_{10}+11\,040\,\gamma_1\,\delta_{10}+529\,920\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-$$
$$5760\,\delta_{10}^2-115\,200\,\gamma_1\,\delta_{10}^2+46\,080\,\delta_{10}^3-9840\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+11\,520\,\delta_{10}\,\delta_{12}+$$
$$\left.15\,360\,\delta_{23}-3280\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+3840\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\right),\ \alpha_6\to 0,\ \alpha_{15}\to 0,$$

$$\alpha_{24}\to 0,\ \alpha_{33}\to 0,\ \alpha_{42}\to 0,\ \alpha_{51}\to 0,\ \beta_6\to\frac{1}{448}\,(-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50}),$$

$$\gamma_6\to 0,\ \gamma_{15}\to 0,\ \gamma_{24}\to 0,$$

$$\beta_{60}\to\frac{1}{42}\,(-1+42\,\gamma_1+210\,\gamma_{30}+126\,\gamma_{50}),$$

$$\beta_{51} \to \frac{1}{2688}(-25+1232\,\gamma_1+6720\,\gamma_{12}+4480\,\gamma_{30}+6720\,\gamma_{41}+1344\,\gamma_{50}),$$

$$\beta_{42} \to \frac{1}{240}(-1+58\,\gamma_1+600\,\gamma_{12}+480\,\gamma_{23}+120\,\gamma_{30}+240\,\gamma_{41}),$$

$$\beta_{33} \to \frac{1}{13\,440}(-47+2100\,\gamma_1+25\,200\,\gamma_{12}+40\,320\,\gamma_{23}+5040\,\gamma_{30}),\ \gamma_{33} \to 0,$$

$$\beta_{24} \to \frac{1}{2880}(-11+300\,\gamma_1+4320\,\gamma_{12}+5760\,\gamma_{23}+1920\,\gamma_{30}+2880\,\gamma_{41}),$$

$$\gamma_{42} \to 0,\ \beta_{15} \to \frac{1}{2688}(-11+224\,\gamma_1+3360\,\gamma_{12}+3360\,\gamma_{30}+6720\,\gamma_{41}+1344\,\gamma_{50}),$$

$$\gamma_{51} \to 0,\ \gamma_{60} \to 0,\ \delta_{15} \to \frac{3-84\,\gamma_1-1120\,\gamma_{30}-1344\,\gamma_{50}}{2688},$$

$$\begin{aligned}\delta_{24} \to \frac{1}{403\,200}\big(&-2161+18\,312\,\gamma_1-241\,920\,\gamma_1^2-4\,838\,400\,\gamma_1^3+201\,600\,\gamma_{12}+\\&3\,870\,720\,\gamma_1\,\gamma_{12}-201\,600\,\gamma_{30}-1\,935\,360\,\gamma_1\,\gamma_{30}-403\,200\,\gamma_{41}-52\,080\,\delta_{10}-\\&1\,209\,600\,\gamma_1\,\delta_{10}-2\,903\,040\,\gamma_1^2\,\delta_{10}-1\,935\,360\,\gamma_{12}\,\delta_{10}+967\,680\,\gamma_{30}\,\delta_{10}+241\,920\,\delta_{10}^2+\\&1\,935\,360\,\gamma_1\,\delta_{10}^2-342\,720\,\delta_{12}-3\,386\,880\,\gamma_1\,\delta_{12}-1\,935\,360\,\delta_{10}\,\delta_{12}+322\,560\,\delta_{23}+\\&87\,360\,\delta_{30}+483\,840\,\gamma_1\,\delta_{30}-645\,120\,\delta_{10}\,\delta_{30}+161\,280\,\delta_{41}-80\,640\,\delta_{50}\big),\end{aligned}$$

$$\delta_{33} \to \frac{1}{3840}(-7+240\,\gamma_1-5760\,\gamma_{23}+960\,\gamma_{30}+5760\,\delta_{23}),$$

$$\begin{aligned}\delta_{42} \to \frac{1}{20\,160}\big(&47-721\,\gamma_1+11\,760\,\gamma_1^2+120\,960\,\gamma_1^3+13\,440\,\gamma_{12}-161\,280\,\gamma_1\,\gamma_{12}-40\,320\,\gamma_{23}+\\&7840\,\gamma_{30}+107\,520\,\gamma_1\,\gamma_{30}+1617\,\delta_{10}+26\,880\,\gamma_1\,\delta_{10}+80\,640\,\gamma_1^2\,\delta_{10}+80\,640\,\gamma_{12}\,\delta_{10}-\\&53\,760\,\gamma_{30}\,\delta_{10}-5040\,\delta_{10}^2+40\,320\,\gamma_1\,\delta_{10}^2+80\,640\,\delta_{10}^3-3360\,\delta_{12}+141\,120\,\gamma_1\,\delta_{12}+\\&20\,160\,\delta_{10}\,\delta_{12}+26\,880\,\delta_{23}-4480\,\delta_{30}-33\,600\,\gamma_1\,\delta_{30}+6720\,\delta_{10}\,\delta_{30}+13\,440\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\delta_{51} \to \frac{1}{4608}\big(&47-2064\,\gamma_1+5760\,\gamma_1^2+11\,520\,\gamma_{12}-11\,520\,\gamma_{41}+240\,\delta_{10}-\\&5760\,\gamma_1\,\delta_{10}-11\,520\,\delta_{10}^2-8640\,\delta_{12}-2880\,\delta_{30}+11\,520\,\delta_{41}\big),\end{aligned}$$

$$\delta_{60} \to \frac{1}{8}(1-24\,\gamma_1+40\,\gamma_{30}-24\,\gamma_{50}+24\,\delta_{10}-40\,\delta_{30}+24\,\delta_{50}),\ \omega_6 \to 0,$$

$$\begin{aligned}\omega_{15} \to \frac{1}{7680}\big(&139-3448\,\gamma_1-30\,720\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+\\&6400\,\gamma_{30}+61\,440\,\gamma_1\,\gamma_{30}+4080\,\delta_{10}-19\,200\,\gamma_1\,\delta_{10}+322\,560\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-\\&30\,720\,\gamma_{30}\,\delta_{10}+11\,520\,\delta_{10}^2+92\,160\,\gamma_1\,\delta_{10}^2-16\,320\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}-\\&92\,160\,\delta_{10}\,\delta_{12}+15\,360\,\delta_{23}-5440\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}-30\,720\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{24} \to \frac{1}{7680}\big(&49-1048\,\gamma_1-19\,200\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+61\,440\\&\gamma_1\,\gamma_{30}+1200\,\delta_{10}+7680\,\gamma_1\,\delta_{10}+506\,880\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-11\,520\,\delta_{10}^2-\\&92\,160\,\gamma_1\,\delta_{10}^2-10\,560\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+15\,360\,\delta_{23}-3520\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{42} \to \frac{1}{7680}\big(&49-1048\,\gamma_1-19\,200\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+61\,440\\&\gamma_1\,\gamma_{30}+1200\,\delta_{10}+7680\,\gamma_1\,\delta_{10}+506\,880\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-11\,520\,\delta_{10}^2-\\&92\,160\,\gamma_1\,\delta_{10}^2-10\,560\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+15\,360\,\delta_{23}-3520\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{51} \to \frac{1}{7680}\big(&139-3448\,\gamma_1-30\,720\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+\end{aligned}$$

$$6400\,\gamma_{30}+61\,440\,\gamma_1\,\gamma_{30}+4080\,\delta_{10}-19\,200\,\gamma_1\,\delta_{10}+322\,560\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}+11\,520\,\delta_{10}^2+92\,160\,\gamma_1\,\delta_{10}^2-16\,320\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}-92\,160\,\delta_{10}\,\delta_{12}+15\,360\,\delta_{23}-5440\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}-30\,720\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\Big),$$

$$\omega_{60}\to 0,\ \delta_{32}\to\frac{1}{5760}\Big(3+17\,\gamma_1+1680\,\gamma_1^2+17\,280\,\gamma_1^3-2400\,\gamma_{12}-23\,040\,\gamma_1\,\gamma_{12}+1600\,\gamma_{30}+15\,360\,\gamma_1\,\gamma_{30}+231\,\delta_{10}+3840\,\gamma_1\,\delta_{10}+11\,520\,\gamma_1^2\,\delta_{10}+11\,520\,\gamma_{12}\,\delta_{10}-7680\,\gamma_{30}\,\delta_{10}-720\,\delta_{10}^2+5760\,\gamma_1\,\delta_{10}^2+11\,520\,\delta_{10}^3+2400\,\delta_{12}+20\,160\,\gamma_1\,\delta_{12}+2880\,\delta_{10}\,\delta_{12}+3840\,\delta_{23}-640\,\delta_{30}-4800\,\gamma_1\,\delta_{30}+960\,\delta_{10}\,\delta_{30}+1920\,\delta_{41}\Big),$$

$$\kappa_6\to\frac{1}{61\,440}\Big(-13+8416\,\gamma_1-72\,960\,\gamma_1^2-5\,529\,600\,\gamma_1^3+153\,600\,\gamma_{12}+1\,474\,560\,\gamma_1\,\gamma_{12}+51\,200\,\gamma_{30}+491\,520\,\gamma_1\,\gamma_{30}+240\,\delta_{10}+249\,600\,\gamma_1\,\delta_{10}+5\,345\,280\,\gamma_1^2\,\delta_{10}-737\,280\,\gamma_{12}\,\delta_{10}-245\,760\,\gamma_{30}\,\delta_{10}-80\,640\,\delta_{10}^2-2\,027\,520\,\gamma_1\,\delta_{10}^2+921\,600\,\delta_{10}^3-44\,160\,\delta_{12}-1\,290\,240\,\gamma_1\,\delta_{12}+645\,120\,\delta_{10}\,\delta_{12}+122\,880\,\delta_{23}-14\,720\,\delta_{30}-430\,080\,\gamma_1\,\delta_{30}+215\,040\,\delta_{10}\,\delta_{30}+61\,440\,\delta_{41}-10\,800\,\kappa_1^2+576\,000\,\gamma_1\,\kappa_1^2+2\,764\,800\,\gamma_1^2\,\kappa_1^2-2\,764\,800\,\gamma_1\,\delta_{10}\,\kappa_1^2+2\,764\,800\,\delta_{10}^2\,\kappa_1^2+1\,382\,400\,\delta_{12}\,\kappa_1^2+460\,800\,\delta_{30}\,\kappa_1^2+57\,600\,\kappa_1^4+921\,600\,\delta_{10}\,\kappa_1^4+61\,440\,\kappa_1^6\Big),$$

$$\delta_{14}\to\frac{1}{57\,600}\Big(-283+936\,\gamma_1-34\,560\,\gamma_1^2-691\,200\,\gamma_1^3+57\,600\,\gamma_{12}+552\,960\,\gamma_1\,\gamma_{12}-28\,800\,\gamma_{30}-276\,480\,\gamma_1\,\gamma_{30}-7440\,\delta_{10}-172\,800\,\gamma_1\,\delta_{10}-414\,720\,\gamma_1^2\,\delta_{10}-276\,480\,\gamma_{12}\,\delta_{10}+138\,240\,\gamma_{30}\,\delta_{10}+34\,560\,\delta_{10}^2+276\,480\,\gamma_1\,\delta_{10}^2-48\,960\,\delta_{12}-483\,840\,\gamma_1\,\delta_{12}-276\,480\,\delta_{10}\,\delta_{12}+46\,080\,\delta_{23}+12\,480\,\delta_{30}+69\,120\,\gamma_1\,\delta_{30}-92\,160\,\delta_{10}\,\delta_{30}+23\,040\,\delta_{41}-11\,520\,\delta_{50}\Big),$$

$$\gamma_{32}\to\frac{1}{20\,160}(-13+420\,\gamma_1-5040\,\gamma_{12}+20\,160\,\gamma_{23}+1680\,\gamma_{30}),$$

$$\gamma_5\to\frac{-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50}}{1344},$$

$$\gamma_{14}\to\frac{3-196\,\gamma_1+3360\,\gamma_{12}+6720\,\gamma_{41}}{6720}\Big\}\Big\}$$

```
{V0, C0} /. sol
```

$$\left\{\left(1+\frac{1}{16}\,c_1\,c_2\,(1+16\,\delta_{10})\,\hbar^2+\left(\frac{1}{256}\,c_1^2\,c_2^2\,\left(-1+40\,\gamma_1+192\,\gamma_1^2-8\,\delta_{10}-192\,\gamma_1\,\delta_{10}+128\,\delta_{10}^2+96\,\delta_{12}+3\right.\right.\right.\right.$$

```
(V0 /. sol) // Rot120 // dη[1] // dP[2 → 1]
```

$$\begin{pmatrix}1-\frac{c_1\hbar}{2}+\frac{7}{48}\,c_1^2\,\hbar^2-\frac{1}{32}\,c_1^3\,\hbar^3+\frac{121\,c_1^4\,\hbar^4}{23\,040}-\frac{11\,c_1^5\,\hbar^5}{15\,360}+\frac{127\,c_1^6\,\hbar^6}{1\,548\,288}+O[\hbar]^7 & h[1]\\ t[1] & -\frac{1}{2}+\frac{c_1\hbar}{8}-\frac{1}{48}\,c_1^2\,\hbar^2+\frac{1}{384}\,c_1^3\,\hbar^3-\frac{c_1^4\,\hbar^4}{3840}-\end{pmatrix}$$

```
(V0 /. sol) // Rot120 // dη[1] // dP[2 → 1] // dA[1] // dcap[1]
```

$$\begin{pmatrix}1-\frac{c_1\hbar}{2}+\frac{7}{48}\,c_1^2\,\hbar^2-\frac{1}{32}\,c_1^3\,\hbar^3+\frac{121\,c_1^4\,\hbar^4}{23\,040}-\frac{11\,c_1^5\,\hbar^5}{15\,360}+\frac{127\,c_1^6\,\hbar^6}{1\,548\,288}+O[\hbar]^7\\ t[1]\end{pmatrix}$$

```
indvars = Union[Flatten[Union[Cases[Last /@ #, ϵ_k_ :> ϵ_k, Infinity]] & /@ {sol}]]
```

$$\{\gamma_1,\gamma_{12},\gamma_{23},\gamma_{30},\gamma_{41},\gamma_{50},\delta_{10},\delta_{12},\delta_{23},\delta_{30},\delta_{41},\delta_{50},\kappa_1\}$$

sol

$$\left\{\alpha_0\to 0,\ \beta_0\to\frac{1}{2},\ \gamma_0\to 0,\ \delta_0\to 0,\ \kappa_0\to 1,\ \omega_0\to 1,\ \alpha_{10}\to 0,\ \beta_{10}\to\frac{1}{8}+\gamma_1,\ \beta_1\to\frac{1}{8}+\frac{1}{24}\ (-1+24\,\gamma_1),\right.$$
$$\delta_1\to 0,\ \alpha_1\to\frac{1}{8}+\delta_{10},\ \omega_1\to 0,\ \omega_{10}\to 0,\ \kappa_2\to\frac{1}{16}\left(1+16\,\delta_{10}+16\,\kappa_1^2\right),\ \alpha_{20}\to 0,\ \delta_2\to 0,$$
$$\omega_{11}\to\frac{1}{16}\ (1+16\,\delta_{10}),\ \alpha_2\to 0,\ \alpha_{11}\to 0,\ \beta_{20}\to\frac{1}{24}\ (1+24\,\gamma_1),\ \beta_2\to\gamma_1,\ \gamma_2\to 0,$$
$$\beta_{11}\to\frac{1}{48}\ (1+48\,\gamma_1),\ \gamma_{11}\to 0,\ \gamma_{20}\to 0,\ \delta_{11}\to-\frac{\gamma_1}{2},\ \delta_{20}\to\frac{1}{24}\ (1-24\,\gamma_1+24\,\delta_{10}),\ \omega_2\to 0,$$
$$\omega_{20}\to 0,\ \gamma_{10}\to\frac{1}{24}\ (-1+24\,\gamma_1),\ \alpha_{30}\to 0,\ \beta_{30}\to\frac{1}{64}\left(1+48\,\gamma_1+\frac{1}{15}\ (-7+240\,\gamma_1+960\,\gamma_{30})\right),$$
$$\beta_{21}\to\frac{1}{192}\ (1+144\,\gamma_1+192\,\gamma_{12}),\ \beta_{12}\to\frac{1}{192}\left(-1+144\,\gamma_1+\frac{1}{5}\ (1-80\,\gamma_1+960\,\gamma_{12})\right),$$
$$\beta_3\to\frac{1}{64}\ (-1+48\,\gamma_1+64\,\gamma_{30}),\ \delta_3\to 0,\ \alpha_{21}\to\frac{1}{192}\ (-1+48\,\gamma_1+192\,\delta_{12}),\ \alpha_{12}\to\frac{1}{192}$$
$$\left(-1+144\,\gamma_1+\frac{1}{12}\left(-7-240\,\gamma_1-1152\,\gamma_1^2-48\,\delta_{10}+1152\,\gamma_1\,\delta_{10}+2304\,\delta_{10}^2+1728\,\delta_{12}+576\,\delta_{30}\right)\right),$$
$$\alpha_3\to\frac{1}{64}\ (-3+96\,\gamma_1-48\,\delta_{10}+64\,\delta_{30}),\ \kappa_3\to\frac{1}{16}\,\kappa_1\left(3+48\,\delta_{10}+16\,\kappa_1^2\right),\ \omega_3\to 0,\ \omega_{12}\to 0,$$
$$\omega_{21}\to 0,\ \omega_{30}\to 0,\ \alpha_{40}\to 0,\ \beta_{40}\to\frac{1}{120}\ (-1+120\,\gamma_1+240\,\gamma_{30}),\ \gamma_4\to 0,\ \delta_4\to 0,$$
$$\omega_{22}\to\frac{1}{64}\left(-1+40\,\gamma_1+192\,\gamma_1^2-8\,\delta_{10}-192\,\gamma_1\,\delta_{10}+128\,\delta_{10}^2+96\,\delta_{12}+32\,\delta_{30}\right),\ \alpha_4\to 0,\ \alpha_{13}\to 0,$$
$$\alpha_{22}\to 0,\ \alpha_{31}\to 0,\ \beta_4\to\frac{1}{480}\ (-7+240\,\gamma_1+960\,\gamma_{30}),\ \beta_{31}\to\frac{1}{384}\ (-1+240\,\gamma_1+576\,\gamma_{12}+192\,\gamma_{30}),$$
$$\gamma_{13}\to 0,\ \beta_{22}\to\frac{1}{960}\ (-3+400\,\gamma_1+1920\,\gamma_{12}),\ \gamma_{22}\to 0,\ \beta_{13}\to\frac{1}{128}\ (-1+48\,\gamma_1+192\,\gamma_{12}+64\,\gamma_{30}),$$
$$\gamma_{31}\to 0,\ \gamma_{40}\to 0,\ \delta_{13}\to\frac{7-240\,\gamma_1-960\,\gamma_{30}}{1920},\ \delta_{22}\to-\gamma_{12}+\delta_{12},\ \delta_{31}\to\frac{1}{7680}$$
$$\left(-47+1680\,\gamma_1-5760\,\gamma_1^2-11\,520\,\gamma_{12}-240\,\delta_{10}+5760\,\gamma_1\,\delta_{10}+11\,520\,\delta_{10}^2+8640\,\delta_{12}+2880\,\delta_{30}\right),$$
$$\delta_{40}\to\frac{1}{24}\ (-1+24\,\gamma_1-48\,\gamma_{30}-24\,\delta_{10}+48\,\delta_{30}),\ \omega_4\to 0,\ \kappa_4\to$$
$$\frac{1}{256}\left(-3+160\,\gamma_1+768\,\gamma_1^2-768\,\gamma_1\,\delta_{10}+768\,\delta_{10}^2+384\,\delta_{12}+128\,\delta_{30}+96\,\kappa_1^2+1536\,\delta_{10}\,\kappa_1^2+256\,\kappa_1^4\right),$$
$$\omega_{13}\to\frac{1}{128}\left(-3+80\,\gamma_1+384\,\gamma_1^2-48\,\delta_{10}-384\,\gamma_1\,\delta_{10}+192\,\delta_{12}+64\,\delta_{30}\right),$$
$$\omega_{31}\to\frac{1}{128}\left(-3+80\,\gamma_1+384\,\gamma_1^2-48\,\delta_{10}-384\,\gamma_1\,\delta_{10}+192\,\delta_{12}+64\,\delta_{30}\right),\ \omega_{40}\to 0,$$
$$\delta_{21}\to\frac{1}{2304}\left(-7-240\,\gamma_1-1152\,\gamma_1^2-48\,\delta_{10}+1152\,\gamma_1\,\delta_{10}+2304\,\delta_{10}^2+1728\,\delta_{12}+576\,\delta_{30}\right),$$
$$\gamma_{21}\to\frac{1}{960}\ (1-80\,\gamma_1+960\,\gamma_{12}),\ \gamma_3\to\frac{1}{960}\ (-7+240\,\gamma_1+960\,\gamma_{30}),$$
$$\beta_{50}\to\frac{1}{64}\left(-1+60\,\gamma_1+160\,\gamma_{30}+\frac{1}{21}\ (-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50})\right),$$

$$\beta_{14}\to\frac{1}{160}\,(-1+30\,\gamma_1+240\,\gamma_{12}+160\,\gamma_{30}+160\,\gamma_{41}),$$

$$\beta_5\to\frac{1}{96}\,(-1+30\,\gamma_1+240\,\gamma_{30}+96\,\gamma_{50}),\ \alpha_{41}\to\frac{1}{480}$$
$$\Big(-1+30\,\gamma_1+240\,\gamma_{30}+\frac{1}{120}\,\big(-283+936\,\gamma_1-34\,560\,\gamma_1^2-691\,200\,\gamma_1^3+57\,600\,\gamma_{12}+552\,960\,\gamma_1\,\gamma_{12}-$$
$$28\,800\,\gamma_{30}-276\,480\,\gamma_1\,\gamma_{30}-7440\,\delta_{10}-172\,800\,\gamma_1\,\delta_{10}-414\,720\,\gamma_1^2\,\delta_{10}-276\,480\,\gamma_{12}\,\delta_{10}+$$
$$138\,240\,\gamma_{30}\,\delta_{10}+34\,560\,\delta_{10}^2+276\,480\,\gamma_1\,\delta_{10}^2-48\,960\,\delta_{12}-483\,840\,\gamma_1\,\delta_{12}-276\,480\,\delta_{10}$$
$$\delta_{12}+46\,080\,\delta_{23}+12\,480\,\delta_{30}+69\,120\,\gamma_1\,\delta_{30}-92\,160\,\delta_{10}\,\delta_{30}+23\,040\,\delta_{41}-11\,520\,\delta_{50}\big)\Big),$$

$$\alpha_{32}\to\frac{1}{320}\,(-1+30\,\gamma_1+240\,\gamma_{12}+160\,\gamma_{30}+320\,\delta_{23}),\ \alpha_{23}\to$$
$$\frac{1}{3840}\Big(1+8640\,\gamma_{12}-2880\,\delta_{12}+\frac{2}{3}\,\big(3+17\,\gamma_1+1680\,\gamma_1^2+17\,280\,\gamma_1^3-2400\,\gamma_{12}-23\,040\,\gamma_1\,\gamma_{12}+$$
$$1600\,\gamma_{30}+15\,360\,\gamma_1\,\gamma_{30}+231\,\delta_{10}+3840\,\gamma_1\,\delta_{10}+11\,520\,\gamma_1^2\,\delta_{10}+11\,520\,\gamma_{12}\,\delta_{10}-$$
$$7680\,\gamma_{30}\,\delta_{10}-720\,\delta_{10}^2+5760\,\gamma_1\,\delta_{10}^2+11\,520\,\delta_{10}^3+2400\,\delta_{12}+20\,160\,\gamma_1\,\delta_{12}+$$
$$2880\,\delta_{10}\,\delta_{12}+3840\,\delta_{23}-640\,\delta_{30}-4800\,\gamma_1\,\delta_{30}+960\,\delta_{10}\,\delta_{30}+1920\,\delta_{41}\big)\Big),$$

$$\alpha_{14}\to\frac{1}{2560}\,\big(9-560\,\gamma_1+1920\,\gamma_1^2+7680\,\gamma_{12}+1280\,\gamma_{30}+80\,\delta_{10}-1920\,\gamma_1\,\delta_{10}-$$
$$3840\,\delta_{10}^2-2880\,\delta_{12}-960\,\delta_{30}+2560\,\delta_{41}\big),$$

$$\alpha_5\to\frac{1}{128}\,(7-160\,\gamma_1+640\,\gamma_{30}+200\,\delta_{10}-320\,\delta_{30}+128\,\delta_{50}),\ \alpha_{50}\to 0,$$

$$\beta_{41}\to\frac{1}{960}\,\Big(-7+540\,\gamma_1+1440\,\gamma_{12}+960\,\gamma_{30}+\frac{1}{7}\,(3-196\,\gamma_1+3360\,\gamma_{12}+6720\,\gamma_{41})\Big),$$

$$\beta_{32}\to\frac{1}{1920}\,(-7+600\,\gamma_1+4320\,\gamma_{12}+1920\,\gamma_{23}+480\,\gamma_{30}),\ \beta_{23}\to$$
$$\frac{1}{1920}\,\Big(-7+360\,\gamma_1+4320\,\gamma_{12}+480\,\gamma_{30}+\frac{2}{21}\,(-13+420\,\gamma_1-5040\,\gamma_{12}+20\,160\,\gamma_{23}+1680\,\gamma_{30})\Big),$$

$$\delta_5\to 0,\ \omega_5\to 0,\ \omega_{14}\to 0,\ \omega_{23}\to 0,\ \omega_{32}\to 0,\ \omega_{41}\to 0,\ \omega_{50}\to 0,$$

$$\kappa_5\to\frac{1}{256}\,\kappa_1\,\big(-15+800\,\gamma_1+3840\,\gamma_1^2-3840\,\gamma_1\,\delta_{10}+3840\,\delta_{10}^2+$$
$$1920\,\delta_{12}+640\,\delta_{30}+160\,\kappa_1^2+2560\,\delta_{10}\,\kappa_1^2+256\,\kappa_1^4\big),\ \alpha_{60}\to 0,\ \delta_6\to 0,$$

$$\omega_{33}\to\frac{1}{7680}\,\big(49-748\,\gamma_1-17\,760\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+$$
$$61\,440\,\gamma_1\,\gamma_{30}+1380\,\delta_{10}+11\,040\,\gamma_1\,\delta_{10}+529\,920\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-$$
$$5760\,\delta_{10}^2-115\,200\,\gamma_1\,\delta_{10}^2+46\,080\,\delta_{10}^3-9840\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+11\,520\,\delta_{10}\,\delta_{12}+$$
$$15\,360\,\delta_{23}-3280\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+3840\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\big),\ \alpha_6\to 0,\ \alpha_{15}\to 0,$$

$$\alpha_{24}\to 0,\ \alpha_{33}\to 0,\ \alpha_{42}\to 0,\ \alpha_{51}\to 0,\ \beta_6\to\frac{1}{448}\,(-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50}),$$

$$\gamma_6\to 0,\ \gamma_{15}\to 0,\ \gamma_{24}\to 0,$$

$$\beta_{60}\to\frac{1}{42}\,(-1+42\,\gamma_1+210\,\gamma_{30}+126\,\gamma_{50}),$$

$$\beta_{51}\to\frac{1}{2688}\,(-25+1232\,\gamma_1+6720\,\gamma_{12}+4480\,\gamma_{30}+6720\,\gamma_{41}+1344\,\gamma_{50}),$$

$$\beta_{42} \to \frac{1}{240}\,(-1+58\,\gamma_1+600\,\gamma_{12}+480\,\gamma_{23}+120\,\gamma_{30}+240\,\gamma_{41}),$$

$$\beta_{33} \to \frac{1}{13\,440}(-47+2100\,\gamma_1+25\,200\,\gamma_{12}+40\,320\,\gamma_{23}+5040\,\gamma_{30}),\ \gamma_{33} \to 0,$$

$$\beta_{24} \to \frac{1}{2880}(-11+300\,\gamma_1+4320\,\gamma_{12}+5760\,\gamma_{23}+1920\,\gamma_{30}+2880\,\gamma_{41}),$$

$$\gamma_{42} \to 0,\ \beta_{15} \to \frac{1}{2688}(-11+224\,\gamma_1+3360\,\gamma_{12}+3360\,\gamma_{30}+6720\,\gamma_{41}+1344\,\gamma_{50}),$$

$$\gamma_{51} \to 0,\ \gamma_{60} \to 0,\ \delta_{15} \to \frac{3-84\,\gamma_1-1120\,\gamma_{30}-1344\,\gamma_{50}}{2688},$$

$$\begin{aligned}\delta_{24} \to \frac{1}{403\,200}\,\big(&-2161+18\,312\,\gamma_1-241\,920\,\gamma_1^2-4\,838\,400\,\gamma_1^3+201\,600\,\gamma_{12}+\\
&3\,870\,720\,\gamma_1\,\gamma_{12}-201\,600\,\gamma_{30}-1\,935\,360\,\gamma_1\,\gamma_{30}-403\,200\,\gamma_{41}-52\,080\,\delta_{10}-\\
&1\,209\,600\,\gamma_1\,\delta_{10}-2\,903\,040\,\gamma_1^2\,\delta_{10}-1\,935\,360\,\gamma_{12}\,\delta_{10}+967\,680\,\gamma_{30}\,\delta_{10}+241\,920\,\delta_{10}^2+\\
&1\,935\,360\,\gamma_1\,\delta_{10}^2-342\,720\,\delta_{12}-3\,386\,880\,\gamma_1\,\delta_{12}-1\,935\,360\,\delta_{10}\,\delta_{12}+322\,560\,\delta_{23}+\\
&87\,360\,\delta_{30}+483\,840\,\gamma_1\,\delta_{30}-645\,120\,\delta_{10}\,\delta_{30}+161\,280\,\delta_{41}-80\,640\,\delta_{50}\big),\end{aligned}$$

$$\delta_{33} \to \frac{1}{3840}(-7+240\,\gamma_1-5760\,\gamma_{23}+960\,\gamma_{30}+5760\,\delta_{23}),$$

$$\begin{aligned}\delta_{42} \to \frac{1}{20\,160}\,\big(&47-721\,\gamma_1+11\,760\,\gamma_1^2+120\,960\,\gamma_1^3+13\,440\,\gamma_{12}-161\,280\,\gamma_1\,\gamma_{12}-40\,320\,\gamma_{23}+\\
&7840\,\gamma_{30}+107\,520\,\gamma_1\,\gamma_{30}+1617\,\delta_{10}+26\,880\,\gamma_1\,\delta_{10}+80\,640\,\gamma_1^2\,\delta_{10}+80\,640\,\gamma_{12}\,\delta_{10}-\\
&53\,760\,\gamma_{30}\,\delta_{10}-5040\,\delta_{10}^2+40\,320\,\gamma_1\,\delta_{10}^2+80\,640\,\delta_{10}^3-3360\,\delta_{12}+141\,120\,\gamma_1\,\delta_{12}+\\
&20\,160\,\delta_{10}\,\delta_{12}+26\,880\,\delta_{23}-4480\,\delta_{30}-33\,600\,\gamma_1\,\delta_{30}+6720\,\delta_{10}\,\delta_{30}+13\,440\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\delta_{51} \to \frac{1}{4608}\,\big(&47-2064\,\gamma_1+5760\,\gamma_1^2+11\,520\,\gamma_{12}-11\,520\,\gamma_{41}+240\,\delta_{10}-\\
&5760\,\gamma_1\,\delta_{10}-11\,520\,\delta_{10}^2-8640\,\delta_{12}-2880\,\delta_{30}+11\,520\,\delta_{41}\big),\end{aligned}$$

$$\delta_{60} \to \frac{1}{8}\,(1-24\,\gamma_1+40\,\gamma_{30}-24\,\gamma_{50}+24\,\delta_{10}-40\,\delta_{30}+24\,\delta_{50}),\ \omega_6 \to 0,$$

$$\begin{aligned}\omega_{15} \to \frac{1}{7680}\,\big(&139-3448\,\gamma_1-30\,720\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+\\
&6400\,\gamma_{30}+61\,440\,\gamma_1\,\gamma_{30}+4080\,\delta_{10}-19\,200\,\gamma_1\,\delta_{10}+322\,560\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-\\
&30\,720\,\gamma_{30}\,\delta_{10}+11\,520\,\delta_{10}^2+92\,160\,\gamma_1\,\delta_{10}^2-16\,320\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}-\\
&92\,160\,\delta_{10}\,\delta_{12}+15\,360\,\delta_{23}-5440\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}-30\,720\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{24} \to \frac{1}{7680}\,\big(&49-1048\,\gamma_1-19\,200\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+61\,440\\
&\gamma_1\,\gamma_{30}+1200\,\delta_{10}+7680\,\gamma_1\,\delta_{10}+506\,880\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-11\,520\,\delta_{10}^2-\\
&92\,160\,\gamma_1\,\delta_{10}^2-10\,560\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+15\,360\,\delta_{23}-3520\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{42} \to \frac{1}{7680}\,\big(&49-1048\,\gamma_1-19\,200\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+6400\,\gamma_{30}+61\,440\\
&\gamma_1\,\gamma_{30}+1200\,\delta_{10}+7680\,\gamma_1\,\delta_{10}+506\,880\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-30\,720\,\gamma_{30}\,\delta_{10}-11\,520\,\delta_{10}^2-\\
&92\,160\,\gamma_1\,\delta_{10}^2-10\,560\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}+15\,360\,\delta_{23}-3520\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}+7680\,\delta_{41}\big),\end{aligned}$$

$$\begin{aligned}\omega_{51} \to \frac{1}{7680}\,\big(&139-3448\,\gamma_1-30\,720\,\gamma_1^2-691\,200\,\gamma_1^3+19\,200\,\gamma_{12}+184\,320\,\gamma_1\,\gamma_{12}+\\
&6400\,\gamma_{30}+61\,440\,\gamma_1\,\gamma_{30}+4080\,\delta_{10}-19\,200\,\gamma_1\,\delta_{10}+322\,560\,\gamma_1^2\,\delta_{10}-92\,160\,\gamma_{12}\,\delta_{10}-\\
&30\,720\,\gamma_{30}\,\delta_{10}+11\,520\,\delta_{10}^2+92\,160\,\gamma_1\,\delta_{10}^2-16\,320\,\delta_{12}-161\,280\,\gamma_1\,\delta_{12}-\end{aligned}$$

$$92\,160\,\delta_{10}\,\delta_{12}+15\,360\,\delta_{23}-5440\,\delta_{30}-53\,760\,\gamma_1\,\delta_{30}-30\,720\,\delta_{10}\,\delta_{30}+7680\,\delta_{41}\Big),$$

$$\omega_{60}\to 0,\ \delta_{32}\to \frac{1}{5760}\Big(3+17\,\gamma_1+1680\,\gamma_1^2+17\,280\,\gamma_1^3-2400\,\gamma_{12}-23\,040\,\gamma_1\,\gamma_{12}+$$
$$1600\,\gamma_{30}+15\,360\,\gamma_1\,\gamma_{30}+231\,\delta_{10}+3840\,\gamma_1\,\delta_{10}+11\,520\,\gamma_1^2\,\delta_{10}+11\,520\,\gamma_{12}\,\delta_{10}-$$
$$7680\,\gamma_{30}\,\delta_{10}-720\,\delta_{10}^2+5760\,\gamma_1\,\delta_{10}^2+11\,520\,\delta_{10}^3+2400\,\delta_{12}+20\,160\,\gamma_1\,\delta_{12}+$$
$$2880\,\delta_{10}\,\delta_{12}+3840\,\delta_{23}-640\,\delta_{30}-4800\,\gamma_1\,\delta_{30}+960\,\delta_{10}\,\delta_{30}+1920\,\delta_{41}\Big),$$

$$\kappa_6\to \frac{1}{61\,440}\Big(-13+8416\,\gamma_1-72\,960\,\gamma_1^2-5\,529\,600\,\gamma_1^3+153\,600\,\gamma_{12}+1\,474\,560\,\gamma_1\,\gamma_{12}+$$
$$51\,200\,\gamma_{30}+491\,520\,\gamma_1\,\gamma_{30}+240\,\delta_{10}+249\,600\,\gamma_1\,\delta_{10}+5\,345\,280\,\gamma_1^2\,\delta_{10}-737\,280\,\gamma_{12}\,\delta_{10}-$$
$$245\,760\,\gamma_{30}\,\delta_{10}-80\,640\,\delta_{10}^2-2\,027\,520\,\gamma_1\,\delta_{10}^2+921\,600\,\delta_{10}^3-44\,160\,\delta_{12}-1\,290\,240\,\gamma_1\,\delta_{12}+$$
$$645\,120\,\delta_{10}\,\delta_{12}+122\,880\,\delta_{23}-14\,720\,\delta_{30}-430\,080\,\gamma_1\,\delta_{30}+215\,040\,\delta_{10}\,\delta_{30}+61\,440\,\delta_{41}-$$
$$10\,800\,\kappa_1^2+576\,000\,\gamma_1\,\kappa_1^2+2\,764\,800\,\gamma_1^2\,\kappa_1^2-2\,764\,800\,\gamma_1\,\delta_{10}\,\kappa_1^2+2\,764\,800\,\delta_{10}^2\,\kappa_1^2+$$
$$1\,382\,400\,\delta_{12}\,\kappa_1^2+460\,800\,\delta_{30}\,\kappa_1^2+57\,600\,\kappa_1^4+921\,600\,\delta_{10}\,\kappa_1^4+61\,440\,\kappa_1^6\Big),$$

$$\delta_{14}\to \frac{1}{57\,600}\Big(-283+936\,\gamma_1-34\,560\,\gamma_1^2-691\,200\,\gamma_1^3+57\,600\,\gamma_{12}+552\,960\,\gamma_1\,\gamma_{12}-$$
$$28\,800\,\gamma_{30}-276\,480\,\gamma_1\,\gamma_{30}-7440\,\delta_{10}-172\,800\,\gamma_1\,\delta_{10}-414\,720\,\gamma_1^2\,\delta_{10}-276\,480\,\gamma_{12}\,\delta_{10}+$$
$$138\,240\,\gamma_{30}\,\delta_{10}+34\,560\,\delta_{10}^2+276\,480\,\gamma_1\,\delta_{10}^2-48\,960\,\delta_{12}-483\,840\,\gamma_1\,\delta_{12}-276\,480\,\delta_{10}\,\delta_{12}+$$
$$46\,080\,\delta_{23}+12\,480\,\delta_{30}+69\,120\,\gamma_1\,\delta_{30}-92\,160\,\delta_{10}\,\delta_{30}+23\,040\,\delta_{41}-11\,520\,\delta_{50}\Big),$$

$$\gamma_{32}\to \frac{1}{20\,160}(-13+420\,\gamma_1-5040\,\gamma_{12}+20\,160\,\gamma_{23}+1680\,\gamma_{30}),$$

$$\gamma_5\to \frac{-3+84\,\gamma_1+1120\,\gamma_{30}+1344\,\gamma_{50}}{1344},$$

$$\gamma_{14}\to \frac{3-196\,\gamma_1+3360\,\gamma_{12}+6720\,\gamma_{41}}{6720}\Big\}$$

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sol1 = Union[
  sol /. Thread[indvars → 0],
  Thread[indvars → 0]
 ]
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$$\left\{\alpha_0\to 0,\ \alpha_1\to \frac{1}{8},\ \alpha_2\to 0,\ \alpha_3\to -\frac{3}{64},\ \alpha_4\to 0,\ \alpha_5\to \frac{7}{128},\ \alpha_6\to 0,\ \alpha_{10}\to 0,\ \alpha_{11}\to 0,\ \alpha_{12}\to -\frac{19}{2304},\right.$$
$$\alpha_{13}\to 0,\ \alpha_{14}\to \frac{9}{2560},\ \alpha_{15}\to 0,\ \alpha_{20}\to 0,\ \alpha_{21}\to -\frac{1}{192},\ \alpha_{22}\to 0,\ \alpha_{23}\to \frac{1}{1280},\ \alpha_{24}\to 0,$$
$$\alpha_{30}\to 0,\ \alpha_{31}\to 0,\ \alpha_{32}\to -\frac{1}{320},\ \alpha_{33}\to 0,\ \alpha_{40}\to 0,\ \alpha_{41}\to -\frac{403}{57\,600},\ \alpha_{42}\to 0,\ \alpha_{50}\to 0,\ \alpha_{51}\to 0,$$
$$\alpha_{60}\to 0,\ \beta_0\to \frac{1}{2},\ \beta_1\to \frac{1}{12},\ \beta_2\to 0,\ \beta_3\to -\frac{1}{64},\ \beta_4\to -\frac{7}{480},\ \beta_5\to -\frac{1}{96},\ \beta_6\to -\frac{3}{448},$$
$$\beta_{10}\to \frac{1}{8},\ \beta_{11}\to \frac{1}{48},\ \beta_{12}\to -\frac{1}{240},\ \beta_{13}\to -\frac{1}{128},\ \beta_{14}\to -\frac{1}{160},\ \beta_{15}\to -\frac{11}{2688},\ \beta_{20}\to \frac{1}{24},$$
$$\beta_{21}\to \frac{1}{192},\ \beta_{22}\to -\frac{1}{320},\ \beta_{23}\to -\frac{173}{40\,320},\ \beta_{24}\to -\frac{11}{2880},\ \beta_{30}\to \frac{1}{120},\ \beta_{31}\to -\frac{1}{384},$$
$$\beta_{32}\to -\frac{7}{1920},\ \beta_{33}\to -\frac{47}{13\,440},\ \beta_{40}\to -\frac{1}{120},\ \beta_{41}\to -\frac{23}{3360},\ \beta_{42}\to -\frac{1}{240},\ \beta_{50}\to -\frac{1}{56},$$
$$\beta_{51}\to -\frac{25}{2688},\ \beta_{60}\to -\frac{1}{42},\ \gamma_0\to 0,\ \gamma_1\to 0,\ \gamma_2\to 0,\ \gamma_3\to -\frac{7}{960},\ \gamma_4\to 0,\ \gamma_5\to -\frac{1}{448},$$
$$\gamma_6\to 0,\ \gamma_{10}\to -\frac{1}{24},\ \gamma_{11}\to 0,\ \gamma_{12}\to 0,\ \gamma_{13}\to 0,\ \gamma_{14}\to \frac{1}{2240},\ \gamma_{15}\to 0,\ \gamma_{20}\to 0,\ \gamma_{21}\to \frac{1}{960},$$
$$\gamma_{22}\to 0,\ \gamma_{23}\to 0,\ \gamma_{24}\to 0,\ \gamma_{30}\to 0,\ \gamma_{31}\to 0,\ \gamma_{32}\to -\frac{13}{20\,160},\ \gamma_{33}\to 0,\ \gamma_{40}\to 0,\ \gamma_{41}\to 0,$$
$$\gamma_{42}\to 0,\ \gamma_{50}\to 0,\ \gamma_{51}\to 0,\ \gamma_{60}\to 0,\ \delta_0\to 0,\ \delta_1\to 0,\ \delta_2\to 0,\ \delta_3\to 0,\ \delta_4\to 0,\ \delta_5\to 0,$$
$$\delta_6\to 0,\ \delta_{10}\to 0,\ \delta_{11}\to 0,\ \delta_{12}\to 0,\ \delta_{13}\to \frac{7}{1920},\ \delta_{14}\to -\frac{283}{57\,600},\ \delta_{15}\to \frac{1}{896},\ \delta_{20}\to \frac{1}{24},$$
$$\delta_{21}\to -\frac{7}{2304},\ \delta_{22}\to 0,\ \delta_{23}\to 0,\ \delta_{24}\to -\frac{2161}{403\,200},\ \delta_{30}\to 0,\ \delta_{31}\to -\frac{47}{7680},\ \delta_{32}\to \frac{1}{1920},$$
$$\delta_{33}\to -\frac{7}{3840},\ \delta_{40}\to -\frac{1}{24},\ \delta_{41}\to 0,\ \delta_{42}\to \frac{47}{20\,160},\ \delta_{50}\to 0,\ \delta_{51}\to \frac{47}{4608},\ \delta_{60}\to \frac{1}{8},\ \kappa_0\to 1,$$
$$\kappa_1\to 0,\ \kappa_2\to \frac{1}{16},\ \kappa_3\to 0,\ \kappa_4\to -\frac{3}{256},\ \kappa_5\to 0,\ \kappa_6\to -\frac{13}{61\,440},\ \omega_0\to 1,\ \omega_1\to 0,\ \omega_2\to 0,$$
$$\omega_3\to 0,\ \omega_4\to 0,\ \omega_5\to 0,\ \omega_6\to 0,\ \omega_{10}\to 0,\ \omega_{11}\to \frac{1}{16},\ \omega_{12}\to 0,\ \omega_{13}\to -\frac{3}{128},\ \omega_{14}\to 0,$$
$$\omega_{15}\to \frac{139}{7680},\ \omega_{20}\to 0,\ \omega_{21}\to 0,\ \omega_{22}\to -\frac{1}{64},\ \omega_{23}\to 0,\ \omega_{24}\to \frac{49}{7680},\ \omega_{30}\to 0,\ \omega_{31}\to -\frac{3}{128},$$
$$\left.\omega_{32}\to 0,\ \omega_{33}\to \frac{49}{7680},\ \omega_{40}\to 0,\ \omega_{41}\to 0,\ \omega_{42}\to \frac{49}{7680},\ \omega_{50}\to 0,\ \omega_{51}\to \frac{139}{7680},\ \omega_{60}\to 0\right\}$$

V1 = V0 /. sol1

$$\left(1+\frac{1}{16}c_1c_2\hbar^2+\left(-\frac{1}{256}c_1^3c_2-\frac{1}{256}c_1^2c_2^2-\frac{1}{256}c_1c_2^3\right)\hbar^4+\left(\frac{139\,c_1^5c_2}{921\,600}+\frac{49\,c_1^4c_2^2}{368\,640}+\frac{49\,c_1^3c_2^3}{276\,480}+\frac{49\,c_1^2c_2^4}{368\,640}+\frac{139\,c_1c_2^5}{921\,600}\right)\right.$$
$$t[1]$$
$$t[2]$$

C1 = C0 /. sol1

$$\left(1+\frac{1}{32}c_1^2\hbar^2-\frac{c_1^4\hbar^4}{2048}-\frac{13\,c_1^6\hbar^6}{44\,236\,800}+O[\hbar]^7\right)$$
$$t[1]$$

```
HardR4[V1]
```

True

```
TwistEq[V1]
```

True

```
V1 ** (V1 // dA[1] // dA[2])
```

(1)

```
CapEquation[V1, C1]
```

True

```
Φ1 = Φ[V1]
```

$$\begin{pmatrix} 1 & \\ t[1] & \left(-\frac{23\,c_1\,c_2\,c_3}{2304}-\frac{11}{576}\,c_2^2\,c_3-\frac{65\,c_2\,c_3^2}{4608}\right)\hbar^3+\left(\frac{179\,c_1^3\,c_2\,c_3}{172\,800}+\frac{1531\,c_1^2\,c_2^2\,c\right. \\ t[2] & -\frac{c_3\,\hbar}{8}+\left(\frac{1}{720}\,c_1^2\,c_3-\frac{1}{576}\,c_1\,c_2\,c_3-\frac{1}{960}\,c_2^2\,c_3-\frac{5\,c_1\,c_3^2}{4608}+\frac{251\,c_2\,c_3^2}{23\,040}+\frac{19\,c_3^3}{2880}\right)\hbar^3+\left(\frac{37\,c_1^4\,c_3}{153\,600}+\frac{4597\,c_1^3\,c_2\,c_3}{4\,838\,400}+\frac{3}{} \\ t[3] & \frac{c_2\,\hbar}{24}+\left(\frac{13\,c_1^2\,c_2}{5760}+\frac{1}{192}\,c_1\,c_2^2-\frac{7\,c_2^3}{5760}+\frac{17\,c_1\,c_2\,c_3}{2880}-\frac{11\,c_2^2\,c_3}{1440}-\frac{7\,c_2\,c_3^2}{1920}\right)\hbar^3+\left(\frac{13\,c_1^4\,c_2}{138\,240}+\frac{151\,c_1^3\,c_2^2}{967\,680}+\frac{11\,c_1^2\,c_2^3}{36\,864}\right. \end{pmatrix}$$

```
Pentagon[Φ1]
```

True

```
Hexagon[+1, Φ1]
```

True

```
Hexagon[-1, Φ1]
```

True

```
Φ1 ** (Φ1 // dP[3, 2, 1])
```

(1)

```
Φ1 ** (Φ1 // dS[1] // dS[2] // dS[3])
```

(1)

```
R[1, 2, 1 / 2]
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{1}{2}+\frac{c_1\,\hbar}{8}+\frac{1}{48}\,c_1^2\,\hbar^2+\frac{1}{384}\,c_1^3\,\hbar^3+\frac{c_1^4\,\hbar^4}{3840}+\frac{c_1^5\,\hbar^5}{46\,080}+\frac{c_1^6\,\hbar^6}{645\,120}+O[\hbar]^7 \end{pmatrix}$$

```
{R[1, 2, 1 / 2], R[1, 2, 1 / 2] // dS[1] // dS[2]}
```

$$\left\{\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{1}{2}+\frac{c_1\,\hbar}{8}+\frac{1}{48}\,c_1^2\,\hbar^2+\frac{1}{384}\,c_1^3\,\hbar^3+\frac{c_1^4\,\hbar^4}{3840}+\frac{c_1^5\,\hbar^5}{46\,080}+\frac{c_1^6\,\hbar^6}{645\,120}+O[\hbar]^7 \end{pmatrix},\right.$$
$$\left.\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{1}{2}+\frac{c_1\,\hbar}{8}+\frac{1}{48}\,c_1^2\,\hbar^2+\frac{1}{384}\,c_1^3\,\hbar^3+\frac{c_1^4\,\hbar^4}{3840}+\frac{c_1^5\,\hbar^5}{46\,080}+\frac{c_1^6\,\hbar^6}{645\,120}+O[\hbar]^7 \end{pmatrix}\right\}$$

```
(R[1, 3, 1 / 2] ** R[2, 3, 1 / 2] ** V1) == (V1 ** (R[1, 3, 1 / 2] // dΔ[1, 1, 2]))
```

$$\frac{1}{2}+\hbar\left(\frac{c_1}{8}-\frac{c_2}{4}\right)+\hbar^2\left(\frac{c_1^2}{48}-\frac{5\,c_1\,c_2}{24}-\frac{5\,c_2^2}{48}\right)+\hbar^3\left(\frac{c_1^3}{384}-\frac{5}{64}\,c_1^2\,c_2-\frac{5}{64}\,c_1\,c_2^2-\frac{c_2^3}{48}\right)+$$
$$\hbar^4\left(\frac{c_1^4}{3840}-\frac{19}{960}\,c_1^3\,c_2-\frac{19}{640}\,c_1^2\,c_2^2-\frac{7}{480}\,c_1\,c_2^3-\frac{3\,c_2^4}{1280}\right)+$$
$$\hbar^5\left(\frac{c_1^5}{46\,080}-\frac{163\,c_1^4\,c_2}{46\,080}-\frac{67\,c_1^3\,c_2^2}{9216}-\frac{233\,c_1^2\,c_2^3}{46\,080}-\frac{c_1\,c_2^4}{1152}+\frac{13\,c_2^5}{46\,080}\right)+$$
$$\hbar^6\left(\frac{c_1^6}{645\,120}-\frac{41\,c_1^5\,c_2}{107\,520}-\frac{233\,c_1^4\,c_2^2}{215\,040}-\frac{319\,c_1^3\,c_2^3}{322\,560}+\frac{c_1^2\,c_2^4}{645\,120}+\frac{43\,c_1\,c_2^5}{107\,520}+\frac{17\,c_2^6}{129\,024}\right)==$$
$$\frac{1}{2}+\hbar\left(\frac{c_1}{8}+\frac{c_2}{8}\right)+\hbar^2\left(\frac{c_1^2}{48}+\frac{c_1\,c_2}{24}+\frac{c_2^2}{48}\right)+\hbar^3\left(\frac{c_1^3}{384}+\frac{1}{128}\,c_1^2\,c_2+\frac{1}{128}\,c_1\,c_2^2+\frac{c_2^3}{384}\right)+$$
$$\hbar^4\left(\frac{c_1^4}{3840}+\frac{1}{960}\,c_1^3\,c_2+\frac{1}{640}\,c_1^2\,c_2^2+\frac{1}{960}\,c_1\,c_2^3+\frac{c_2^4}{3840}\right)+$$
$$\hbar^5\left(\frac{c_1^5}{46\,080}+\frac{c_1^4\,c_2}{9216}+\frac{c_1^3\,c_2^2}{4608}+\frac{c_1^2\,c_2^3}{4608}+\frac{c_1\,c_2^4}{9216}+\frac{c_2^5}{46\,080}\right)+$$
$$\hbar^6\left(\frac{c_1^6}{645\,120}+\frac{c_1^5\,c_2}{107\,520}+\frac{c_1^4\,c_2^2}{43\,008}+\frac{c_1^3\,c_2^3}{32\,256}+\frac{c_1^2\,c_2^4}{43\,008}+\frac{c_1\,c_2^5}{107\,520}+\frac{c_2^6}{645\,120}\right)\;\&\&$$
$$\frac{1}{2}+\hbar\left(\frac{c_1}{2}+\frac{c_2}{8}\right)+\hbar^2\left(\frac{13\,c_1^2}{48}+\frac{c_1\,c_2}{6}+\frac{c_2^2}{48}\right)+\hbar^3\left(\frac{17\,c_1^3}{192}+\frac{3}{32}\,c_1^2\,c_2+\frac{1}{32}\,c_1\,c_2^2+\frac{c_2^3}{384}\right)+$$
$$\hbar^4\left(\frac{27\,c_1^4}{1280}+\frac{31}{960}\,c_1^3\,c_2+\frac{11}{640}\,c_1^2\,c_2^2+\frac{7\,c_1\,c_2^3}{1920}+\frac{c_2^4}{3840}\right)+$$
$$\hbar^5\left(\frac{169\,c_1^5}{46\,080}+\frac{35\,c_1^4\,c_2}{4608}+\frac{253\,c_1^3\,c_2^2}{46\,080}+\frac{11\,c_1^2\,c_2^3}{9216}-\frac{7\,c_1\,c_2^4}{46\,080}+\frac{c_2^5}{46\,080}\right)+$$
$$\hbar^6\left(\frac{253\,c_1^6}{645\,120}+\frac{1}{896}\,c_1^5\,c_2+\frac{673\,c_1^4\,c_2^2}{645\,120}+\frac{17\,c_1^3\,c_2^3}{322\,560}-\frac{79\,c_1^2\,c_2^4}{215\,040}-\frac{13\,c_1\,c_2^5}{107\,520}+\frac{c_2^6}{645\,120}\right)==$$
$$\frac{1}{2}+\hbar\left(\frac{c_1}{8}+\frac{c_2}{8}\right)+\hbar^2\left(\frac{c_1^2}{48}+\frac{c_1\,c_2}{24}+\frac{c_2^2}{48}\right)+\hbar^3\left(\frac{c_1^3}{384}+\frac{1}{128}\,c_1^2\,c_2+\frac{1}{128}\,c_1\,c_2^2+\frac{c_2^3}{384}\right)+$$
$$\hbar^4\left(\frac{c_1^4}{3840}+\frac{1}{960}\,c_1^3\,c_2+\frac{1}{640}\,c_1^2\,c_2^2+\frac{1}{960}\,c_1\,c_2^3+\frac{c_2^4}{3840}\right)+$$
$$\hbar^5\left(\frac{c_1^5}{46\,080}+\frac{c_1^4\,c_2}{9216}+\frac{c_1^3\,c_2^2}{4608}+\frac{c_1^2\,c_2^3}{4608}+\frac{c_1\,c_2^4}{9216}+\frac{c_2^5}{46\,080}\right)+$$
$$\hbar^6\left(\frac{c_1^6}{645\,120}+\frac{c_1^5\,c_2}{107\,520}+\frac{c_1^4\,c_2^2}{43\,008}+\frac{c_1^3\,c_2^3}{32\,256}+\frac{c_1^2\,c_2^4}{43\,008}+\frac{c_1\,c_2^5}{107\,520}+\frac{c_2^6}{645\,120}\right)$$

```
V1 ** (V1 // dS[1] // dS[2])
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & 1+\frac{c_1\hbar}{2}+\frac{1}{6}\,c_1^2\,\hbar^2+\frac{1}{24}\,c_1^3\,\hbar^3+\frac{1}{120}\,c_1^4\,\hbar^4+\frac{1}{720}\,c_1^5\,\hbar^5+\frac{c_1^6\,\hbar^6}{5040}+O[\hbar]^7 \end{pmatrix}$$

```
R[1, 2]
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & 1+\frac{c_1\hbar}{2}+\frac{1}{6}c_1^2\hbar^2+\frac{1}{24}c_1^3\hbar^3+\frac{1}{120}c_1^4\hbar^4+\frac{1}{720}c_1^5\hbar^5+\frac{c_1^6\hbar^6}{5040}+O[\hbar]^7 \end{pmatrix}$$

```
False && Put[{V1, sol}, "VToDegree4-120420.m"]
```

```
False
```

```
False && Put[{V1, C1, sol}, "SolutionToDegree6-120501.m"]
```