Pensieve header: Finding a VC pair with C=1, some "Group Like" tests.

In[1]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.0"];
<< betaCalculus.m
Clear[ℏ]; Unprotect[C];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
    Series[Normal[expr], {ℏ, 0, $PerturbativeDegree}],
    sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
    βSimplify[ω],
    βSimplify[Series[Log[1 + Coefficient[V[[2]] /. t[i_] :> ℏ c_i, #]], {ℏ, 0, 3}]] & /@
      {h[1], h[2]}
  ];
{V, C, sol} = Get["SolutionToDegree6-120501.m"];
{V, C, sol1} = Get["SolutionToDegree6-120518.m"];
{V, C} = {
  βCollect[
    B[ω[ℏ c_1, ℏ c_2], α[ℏ c_1, ℏ c_2] t[1] h[1] +
      β[ℏ c_1, ℏ c_2] t[1] h[2] + γ[ℏ c_1, ℏ c_2] t[2] h[1] + δ[ℏ c_1, ℏ c_2] t[2] h[2]]
  ] /. {
    (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ_0,
    (ϵ : (α | β | γ | δ | ω | κ))^(k__)[___] :> ϵ_FromDigits[{k}]
  },
  βCollect[B[κ[ℏ c_1], 0]] /. {
    (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ_0,
    (ϵ : (α | β | γ | δ | ω | κ))^(k__)[___] :> ϵ_FromDigits[{k}]
  }
} /. sol /. sol1
```

Out[9]= {

In[10]:=
```
{"R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
 "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
 "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
 "VerticalFlipEquation" → V ** (V // dS[1] // dS[2]) == R[1, 2],
 "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
   (C ** (C // dP[12]) // dcap[1] // dcap[2]),
 "SidesNonDegeneracy" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
 "CapsAndCups" → Simplify[C == (C // dS[1])]}
```

Out[10]= {R4 → True, TwistEq → True, Unitarity → True, VerticalFlipEquation → True, CapEquation → True, SidesNonDegeneracy → True, CapsAndCups → True}

In[11]:= C

Out[11]= $\left(1 + c_1 \kappa_1 \hbar + \frac{1}{32} c_1^2 (1 + 16 \delta_{10} + 16 \kappa_1^2) \hbar^2 + \frac{1}{96} c_1^3 \kappa_1 (3 + 48 \delta_{10} + 16 \kappa_1^2) \hbar^3 + \frac{c_1^4 (3 - 768 \delta_{10}^2 + 160 (\frac{1}{12} + \delta_{10}) - 768 \delta_{10} (\ldots}{\ldots} \ldots\right)$

In[12]:= C ** Inverse[C]

Out[12]= (1)

In[13]:= 1 + 1

Out[13]= 2

In[14]:= dP[12][C] ** Inverse[C] ** Inverse[dP[2][C]]

Out[14]= $\left(1 + \left(\frac{c_1 c_2}{16} + c_1 c_2 \delta_{10}\right) \hbar^2 + \left(\frac{1}{72} c_1^3 c_2 + \frac{35 c_1^2 c_2^2}{1536} + \frac{1}{72} c_1 c_2^3 + \frac{1}{16} c_1^2 c_2^2 \delta_{10} - c_1^3 c_2 \delta_{10}^2 - c_1^2 c_2^2 \delta_{10}^2 - c_1 c_2^3 \delta_{10}^2 + \frac{1}{3} \ldots\right.\right.$

In[15]:= sol2 = PerturbativeSolveAlways[C == B[1, 0], ℏ, 6, {c_1}]

Out[15]= $\left\{\kappa_1 \to 0,\ \delta_{10} \to -\frac{1}{16},\ \delta_{30} \to -\frac{23}{768},\ \gamma_{12} \to -\frac{53}{28800} - \frac{8\,\delta_{23}}{15} - \frac{4\,\delta_{41}}{15}\right\}$

In[16]:= βCollect[C /. sol2]

Out[16]= (1)

In[17]:= V

Out[17]= $\left(1 + \frac{1}{16} c_1 c_2 (1 + 16 \delta_{10}) \hbar^2 + \left(\frac{1}{256} c_1^2 c_2^2 \left(-1 - 8 \delta_{10} + 128 \delta_{10}^2 + 40 \left(\frac{1}{12} + \delta_{10}\right) - 192 \delta_{10} \left(\frac{1}{12} + \delta_{10}\right) + 192 \left(\frac{1}{12} \ldots\right.\right.\right.\right.$

In[18]:= βCollect[V /. sol2]

Out[18]= $\left(1 + \ldots \frac{c_2 \hbar}{16} + \left(-\frac{5 c_1^2 c_2}{4608} + \frac{c_1 c_2^2}{4608} + \frac{c_2^3}{4608}\right) \hbar^3 + \left(\frac{c_1^4 c_2}{40960} - \frac{53 c_1^3 c_2^2}{460800} \ldots\right.\right.$
$\ldots -\frac{73 c_1^2 c_2}{2560} - \frac{53 c_1 c_2^2}{57600} + \frac{c_2^3}{23040} - \frac{4}{15} c_1^2 c_2 \delta_{23} - \frac{4}{15} c_1 c_2^2 \delta_{23} - \frac{2}{15} c_1^2 c_2 \delta_{41} - \frac{2}{15} c_1 c_2^2 \ldots$

In[19]:= (V /. sol2) ** Inverse[V /. sol2]

Out[19]= (1)

In[20]:= Inverse[V /. sol2] ** (V /. sol2)

Out[20]= (1)

In[29]:= σ

Out[29]= $\left\{\frac{c_1 c_2}{12} + \frac{1}{48} c_1^3 \kappa_1 (3 + 48 \delta_{10} + 16 \kappa_1^2) \hbar^3 + \frac{1}{46080} c_1^5 \kappa_1 (1365 - 11520 \delta_{10}^2 + 7680 \delta_{30} + 480 \kappa_1^2 + 768 \kappa_1^4 + 480 \delta_{10} (3 + 16 \kappa_1^2)) \hbar^5 + O[\hbar]^7 == 0\right\}$

$\left\{\frac{c_1 c_2}{12} + 2 c_1 c_2 \delta_{10} + \frac{c_2^2}{12} + c_2^2 \delta_{10} + 2 c_1 c_2 \delta_{10} \ldots \right) \hbar^2 + O[\hbar]^7\}$

In[10]:=
```
{
 "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
 "TwistEq" → V ** θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
 "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
 "VerticalFlipEquation" → V ** (V // dS[1] // dS[2]) == R[1, 2],
 "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
   (C * (C // dP[2]) // dcap[1] // dcap[2]),
 "SidesNonDegeneracy" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
 "CapsAndCups" → Simplify[C == (C // dS[1])]
}
```

Out[10]=
$$\left\{\text{R4} \to \text{True},\ \text{TwistEq} \to \text{True},\ \text{Unitarity} \to \text{True},\ \text{VerticalFlipEquation} \to \text{True},\ \text{CapEquation} \to \text{True},\ \text{SidesNonDegeneracy} \to \text{True},\ \text{CapsAndCups} \to 2\,c_1\,\kappa_1\,\hbar + \frac{1}{48}\,c_1^3\,\kappa_1\left(3 + 48\,\delta_{10} + 16\,\kappa_1^2\right)\hbar^3 + \frac{1}{46\,080}\,c_1^5\,\kappa_1\left(365 - 11\,520\,\delta_{10}^2 + 7680\,\delta_{30} + 480\,\kappa_1^2 + 768\,\kappa_1^4 + 480\,\delta_{10}\left(3 + 16\,\kappa_1^2\right)\right)\hbar^5 + O[\hbar]^7 == 0\right\}$$

In[11]:=
```
C
```

Out[11]=
$$\left(1 + c_1\,\kappa_1\,\hbar + \frac{1}{32}\,c_1^2\,(1 + 16\,\delta_{10} + 16\,\kappa_1^2)\,\hbar^2 + \frac{1}{96}\,c_1^3\,\kappa_1\,(3 + 48\,\delta_{10} + 16\,\kappa_1^2)\,\hbar^3 + \frac{c_1^4\left(-3+768\,\delta_{10}^2+160\left(\frac{1}{12}+\delta_{10}\right)-768\,\delta_{10}\left(\right.\right.}{}\right.$$

In[12]:=
```
C ** Inverse[C]
```

Out[12]=
$$(1)$$

In[13]:=
```
1 + 1
```

Out[13]=
$$2$$

In[14]:=
```
dP[12][C] ** Inverse[C] ** Inverse[dP[2][C]]
```

Out[14]=
$$\left(1 + \left(\frac{c_1\,c_2}{16} + c_1\,c_2\,\delta_{10}\right)\hbar^2 + \left(\frac{1}{72}\,c_1^3\,c_2 + \frac{35\,c_1^2\,c_2^2}{1536} + \frac{1}{72}\,c_1\,c_2^3 + \frac{1}{16}\,c_1^2\,c_2^2\,\delta_{10} - c_1^3\,c_2\,\delta_{10}^2 - c_1^2\,c_2^2\,\delta_{10}^2 - c_1\,c_2^3\,\delta_{10}^2 + \frac{1}{3}\,c\right.$$

In[15]:=
```
sol2 = PerturbativeSolveAlways[C == B[1, 0], ħ, 6, {c1}]
```

Out[15]=
$$\left\{\kappa_1 \to 0,\ \delta_{10} \to -\frac{1}{16},\ \delta_{30} \to -\frac{23}{768},\ \gamma_{12} \to -\frac{53}{28\,800} - \frac{8\,\delta_{23}}{15} - \frac{4\,\delta_{41}}{15}\right\}$$

In[16]:=
```
βCollect[C /. sol2]
```

Out[16]=
$$(1)$$

In[17]:=
```
V
```

Out[17]=
$$\left(1 + \frac{1}{16}\,c_1\,c_2\,(1 + 16\,\delta_{10})\,\hbar^2 + \left(\frac{1}{256}\,c_1^2\,c_2^2\left(-1 - 8\,\delta_{10} + 128\,\delta_{10}^2 + 40\left(\frac{1}{12} + \delta_{10}\right) - 192\,\delta_{10}\left(\frac{1}{12} + \delta_{10}\right) + 192\left(\frac{1}{12}\right.\right.\right.$$