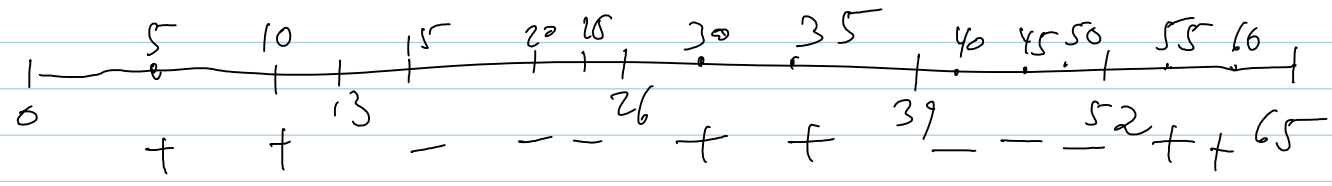
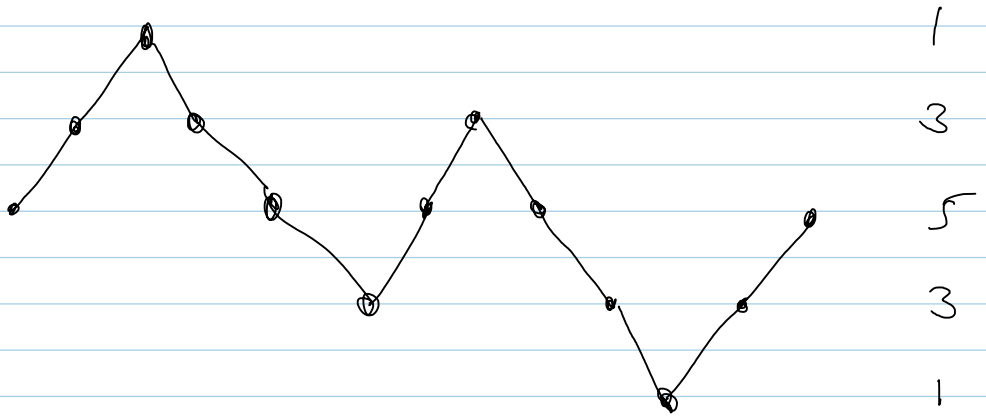
Hoste: Twisted Alexander Polynomials of 2-Bridge Knots

March-15-12
10:03 AM

With Pat Shanahan of LMU

5/13





$$\Delta_{K_{5/13}} = t^{-2} - 3t^{-1} + 5 - 3t + t^2$$

Goal Jazz this up to twisted Alexander polynomial.

2-bridge knots $0 < p < q$, p, q odd, $\gcd(p, q) = 1$

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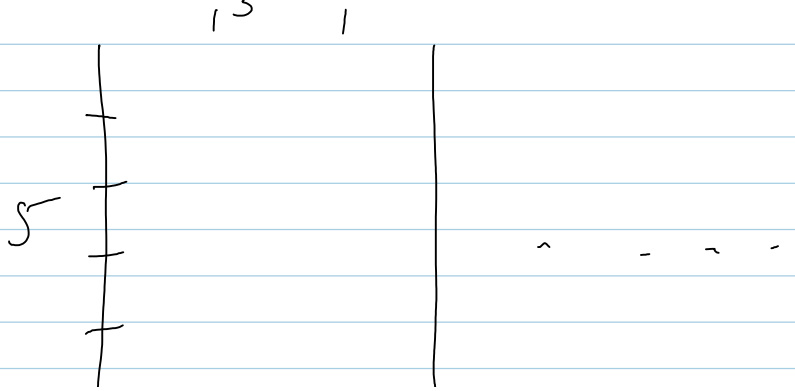

Fundamental group:

$\langle a, b : aw = wb \rangle \quad w = b^{\epsilon_1} a^{\epsilon_2} b^{\epsilon_3} \cdots a^{\epsilon_{q-1}}$

$\epsilon_i = (-1)^{\left\lfloor \frac{ip}{q} \right\rfloor}$

Conjecture (Murasugi–Hirasawa):

Given $\varphi : \pi_1(S^3 - K) \to GL(n, \mathbb{C})$

$\tilde{\Delta}_{K,\varphi}(t)$

↑ twisted Alexander

$\tilde{\Delta}_{K_{p,q},\varphi} = \frac{\Delta_K}{1-t} F(t) F(-t)$

↑ dihedral rep.

↑ Poly w/ $\mathbb{Z}$ coeffs.

where

$F \equiv \left(\frac{\Delta}{1+t}\right)^m \quad \text{mod } n$ 10:20

Δ: $a \mapsto t$, $b \mapsto t$, strike a column, get matrix A, take $\det = \det(A)$

$\tilde{\Delta}$: $a \mapsto t\varphi(a)$, $b \mapsto t\varphi(b)$

Wada, 15–20 years ago.

$a \longmapsto T\psi(a)$

$b \longmapsto T\psi(b)$

strike a super-column got $\widetilde{A}$

$\widetilde{\Delta} = \det(\widetilde{A}) / \det(T\psi(b) - I)$