Poly-only formulas

February-16-12

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Implementation in 2012-04/ControlledSums.nb

Seems to fail, but maybe some "dark front" approach will work?

$$\beta(X,Y)=\left\{\begin{array}{c|ccc} w & x_1 & x_2 & \cdots \\ \hline y_1 & \alpha_{11} & \alpha_{12} & \cdots \\ \vdots & \vdots & & \\ \hline & \sigma_1 & \sigma_2 & \cdots \end{array} : \begin{array}{l} w,\alpha_{ij}\in\mathbb{Z}[T_i^{\pm 1}] \\ \sigma_j \text{ are monomials in } T_i^{\pm 1} \\ \sum_i \alpha_{ij}=(\sigma_j-1)w \quad (\checkmark \end{array}\right\}$$

i.e., $\sigma_j = 1+\frac{1}{w}\sum \alpha_{ij}$

$$\begin{array}{c|c} w & \text{—} \\ \hline x & \text{—}\ \alpha\ \text{—} \\ y & \text{—}\ \beta\ \text{—} \\ \vdots & M \\ \hline & \sigma \end{array} \quad \xmapsto{tm^{xy}_z} \quad \begin{array}{c|c} w & \text{—} \\ \hline z & \text{—}\ \alpha+\beta\ \text{—} \\ \hline \vdots & M \\ \hline & \sigma \end{array}$$

$$\begin{array}{c|ccc} w & x & y & \cdots \\ \hline \vdots & \alpha & \beta & M \\ \hline & \sigma_x & \sigma_y & \sigma' \end{array} \quad \xmapsto{hm^{xy}_z} \quad \begin{array}{c|cc} w & z & \cdots \\ \hline \vdots & \gamma & M \\ \hline & \sigma_x\cdot\sigma_y & \sigma' \end{array} \qquad \gamma=\alpha+\sigma_x\beta$$

$|\gamma| = |\alpha| + \sigma_x|\beta| = (\sigma_x-1)w+\sigma_x(\sigma_y-1)w = (\sigma_x\sigma_y-1)\,w$ ✓

$$\begin{array}{c|c|c} w & x & \cdots \\ \hline y & \alpha & \beta \\ \hline \vdots & \gamma & \delta \\ \hline & & \sigma \end{array} \quad \xmapsto{Swap^{th}_{yx}} \quad \begin{array}{c|c|c} w+\alpha & x & \cdots \\ \hline y & \sigma_x\alpha & \sigma_x\beta \\ \hline \vdots & \gamma & [(w+\alpha)\delta-\gamma\cdot\beta]/w \ ? \\ \hline & & \sigma \end{array}$$

$\sigma_x\alpha+|\gamma| = \left|{\alpha \atop \gamma}\right| + (\sigma_x-1)\alpha = (\sigma_x-1)w+(\sigma_x-1)\alpha = (\sigma_x-1)(w+\alpha)$ ✓

$$\begin{array}{c|c|c} w & x & - \\ \hline y & \alpha & \beta \\ \hline 1 & \gamma & \delta \end{array} \xrightarrow{thswap^x_y} \begin{array}{c|c|c} (1+\alpha)w & & \\ \hline y & \frac{1+|\frac{\alpha}{\gamma}|}{1+\alpha} & (\alpha \quad \beta) \\ \hline 1 & \frac{\gamma}{1+\alpha} & \delta - \frac{\gamma \cdot \beta}{1+\alpha} \end{array}$$

"σ_x"

$$R^+_{xy} = \begin{array}{c|c} 1 & y \\ \hline x & T_x - 1 \end{array} \qquad R^-_{xy} = \begin{array}{c|c} 1 & y \\ \hline x & T_x^{-1} - 1 \end{array}$$

$$1 + \frac{\alpha'}{w} = \frac{w+\alpha'}{w} = 1+\alpha$$

$$\frac{\delta'}{w} = \frac{(1+\alpha)\delta'}{(1+\alpha)w}$$

$$\frac{\sigma_x}{1+\alpha}(\alpha \quad \beta) \longmapsto \frac{\sigma_x}{1+\alpha'/w}\left(\alpha'/w \quad \beta/w\right) =$$

$$= \frac{\sigma_x}{w+\alpha'}(\alpha' \quad \beta') =$$

$$\delta - \frac{\gamma \cdot \beta}{1+\alpha} = \frac{\delta(1+\alpha) - \gamma \cdot \beta}{1+\alpha} = \frac{w\delta(1+\alpha) - w\gamma \cdot \beta}{w(1+\alpha)}$$

$$w\delta(1+\alpha) - w\gamma \cdot \beta \longrightarrow w\frac{\delta}{w}\left(1+\frac{\alpha}{w}\right) - w\frac{\gamma}{w}\frac{\beta}{w}$$

$$= \delta\frac{w+\alpha}{w} - \frac{1}{w}\gamma \cdot \beta$$

$$\frac{(w+\alpha)\delta - \gamma \cdot \beta}{w+\alpha} = \delta - \qquad \frac{\gamma/w}{1+\alpha/w} = \frac{\gamma}{w+\alpha} \longrightarrow \gamma$$

$$\frac{\delta}{w} - \frac{\gamma \cdot \beta}{w^2(1+\alpha/w)} = \frac{1}{w}\left(\delta - \frac{\gamma \cdot \beta}{w+\alpha}\right) \longrightarrow \frac{1}{w}\left((w+\alpha)\delta - \gamma \cdot \beta\right)$$