Formulas after $c\to T$

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$$\begin{array}{c|c} w & — \\ \hline x & — \alpha — \\ y & — \beta \to \\ 1 & M \end{array} \xrightarrow{tm_z^{xy}} \begin{array}{c|c} w & — \\ \hline z & — \alpha+\beta — \\ \hline 1 & M \end{array}$$

$$\begin{array}{c|ccc} w & x & y & — \\ \hline 1 & \alpha & \beta & M \end{array} \xrightarrow{hm_z^{xy}} \begin{array}{c|cc} w & z & — \\ \hline 1 & \gamma & M \end{array}$$

$\gamma = \alpha + \beta(1+|\alpha|)$

where $|\alpha| := \sum \alpha_i$

$$\begin{array}{c|c|c} w & x & — \\ \hline y & \alpha & \beta \\ \hline 1 & \gamma & \delta \end{array} \xrightarrow{Sw^{th}_{yx}} \begin{array}{c|c|c} (1+\alpha)w & x & \\ \hline y & \frac{1+|\alpha|}{1+\alpha} & (\alpha \quad \beta) \\ \hline 1 & \frac{\gamma}{1+\alpha} & \delta - \frac{\gamma\cdot\beta}{1+\alpha} \end{array}$$

"σ_x"

$$R^+_{xy} = \begin{array}{c|c} 1 & y \\ \hline x & T_x - 1 \end{array} \qquad R^-_{xy} = \begin{array}{c|c} 1 & y \\ \hline x & T_x^{-1} - 1 \end{array}$$

$$1+\frac{\alpha'}{w} = \frac{w+\alpha'}{w} = 1+\alpha$$

$$\frac{\delta'}{w} = \frac{(1+\alpha)\delta'}{(1+\alpha)w}$$

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tm[x_, y_, z_][β_] := β /. {t[x] → t[z], t[y] → t[z], T_x → T_z, T_y → T_z};
hm[x_, y_, z_][B[ω_, μ_]] := Module[
   {γx = D[μ, h[x]], γy = D[μ, h[y]], M = μ /. h[x] | h[y] → 0},
   B[ω, M + h[z] (γx + γy + (γx /. t[i_] ⇸ 1) γy)] // βCollect
  ];
swap[x_, y_][B[ω_, μ_]] := Module[
   {α, β, γ, δ, ϵ},
   α = Coefficient[μ, h[x] t[y]];
   β = D[μ, t[y]] /. h[x] → 0;
   γ = D[μ, h[x]] /. t[y] → 0;
   δ = μ /. h[x] | t[y] → 0;
   ϵ = 1 + α;
   B[ω ⋆ ϵ, Plus[
      α (1 + (γ /. t[i_] ⇸ 1) / ϵ) h[x] t[y],
      β (1 + (γ /. t[i_] ⇸ 1) / ϵ) t[y],
      γ / ϵ h[x],
      δ - (1 / ϵ) γ ⋆ β
     ]] // βCollect
  ];
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From 120214 calculator.nb

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R[x_, y_] := B[1, (T_x - 1) t[x] h[y]];
Rinv[x_, y_] := B[1, (1 / T_x - 1) t[x] h[y]];
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$$\alpha \longmapsto \alpha\left(1+\frac{|\gamma|}{1+\alpha}\right)=\frac{\alpha}{1+\alpha}\left(1+\alpha+|\gamma|\right)=\frac{\alpha}{1+\alpha}\left(1+\left|{\alpha \atop \gamma}\right|\right)$$

$$\beta \longmapsto \frac{\beta}{1+\alpha}\left(1+\left|{\alpha \atop \gamma}\right|\right)$$

$$\gamma \longmapsto \frac{\gamma}{1+\alpha}$$

$$\delta \longmapsto \delta - \frac{\gamma\cdot\beta}{1+\alpha}$$

Claim. In $\left(\begin{array}{c|c} \frac{1+\left|{\alpha \atop \gamma}\right|}{1+\alpha}(\alpha & \beta) \\ \hline \frac{\gamma}{1+\alpha} & \delta-\frac{\gamma\cdot\beta}{1+\alpha}\end{array}\right)$ column sums

are as in $\begin{pmatrix}\alpha & \beta \\ \gamma & \delta\end{pmatrix}$.

$$(1+\alpha)(\alpha+|\gamma|)$$
$$\shortparallel$$

<u>Proof</u> 1. $\frac{1+\left|{\alpha \atop \gamma}\right|}{1+\alpha}\alpha+\frac{|\gamma|}{1+\alpha}=\frac{(1+\alpha+|\gamma|)\alpha+|\gamma|}{1+\alpha}=$

$$=\alpha+|\gamma|=\left|{\alpha \atop \gamma}\right|$$

2. $\frac{1+\alpha+\cancel{|\gamma|}}{1+\alpha}\beta + |\delta| - \cancel{\frac{|\gamma|\beta}{1+\alpha}} = \beta + |\delta| = \left|\begin{matrix}\beta \\ \delta\end{matrix}\right|$ $\square$