# $\beta$-Alexander Direct

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9:56 AM

**Tail Action.**

was $tm: \begin{array}{c} x \\ y \end{array}\left(\begin{array}{c} -\alpha- \\ -\beta- \\ M \end{array}\right) \mapsto {}^z\left(\begin{array}{c} -\alpha+\beta- \\ M \end{array}\right)$ , $C_x, C_y \to C_z$

stays the same.

**Head Action.**

was $hm: \left(\begin{array}{cc|c} \alpha & \beta & M \\ x & y & \end{array}\right) \mapsto \left(\begin{array}{c|c} \overset{\gamma}{\overbrace{\alpha+\beta+(C\cdot\alpha)\beta}} & M \\ z & \end{array}\right)$

with $\alpha=\alpha'/w$, $\beta=\beta'/w$ $\gamma=\gamma'/W$ becomes

$hm: \left(\begin{array}{ccc} \alpha' & \beta' & M \\ x & y & \end{array}\right) \mapsto \Big($

as $\gamma'=W\gamma=W(\alpha+\beta+(C\cdot\alpha)\beta)=\alpha'+\beta'+\frac{1}{W}(C\cdot\alpha')\beta'$.

Guess: $C\cdot\alpha'$ is manageable by the linking-matrix meta-group.

$C_y\alpha+\frac{(C_r\cdot\gamma)(C_y\alpha)+C_r\gamma}{1+C_y\alpha}$

$= C\cdot\begin{pmatrix}\alpha\\ \gamma\end{pmatrix}$

$C_y\beta+C_r\delta+\cancel{\frac{(C_r\gamma)C_y\beta}{1+C_y\alpha}}$ – same

**The Swap.**

was $\begin{array}{c} y \\ r \end{array}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}_{x} \longmapsto \begin{pmatrix} \alpha & \beta \\ 0 & \delta \end{pmatrix} +$

$+\frac{1}{1+C_y\cdot\alpha}\begin{pmatrix} (C_r\cdot\gamma)\alpha & (C_r\cdot\gamma)\beta \\ \gamma & -C_y\gamma\cdot\beta \end{pmatrix}$

with $W\mapsto W_{new}=W\cdot(1+C_y\alpha)$

(so if $\alpha = \alpha'/w$, then $w \mapsto w_{new} = w \cdot \left(1 + c_y \frac{\alpha'}{w}\right)$

$$\simeq w + c_y \alpha'$$

$a-1+b-1+(a-1)(b-1) = ab-1$