Burda@GSS: How to solve a quintic polynomial?

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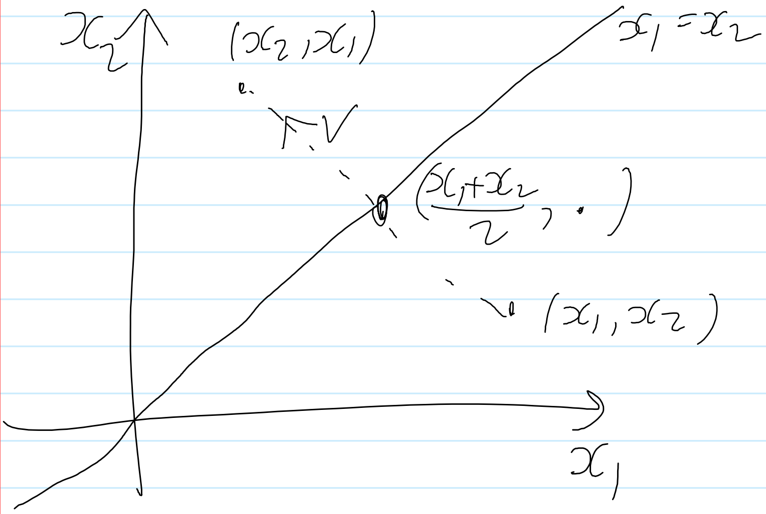
$$x^n + a_1 x^{n-1} + \dots + a_n = 0$$

$$x_1 = f_1(a_1, \dots, a_n) \quad \dots \qquad n \text{ sol'ns.}$$

Every rational function of $x_1 \dots x_n$ which is symmetric is a formula in $a_1 \dots a_n$

Example



$$V = \left(\frac{x_1 - x_2}{2}, \frac{x_2 - x_1}{2}\right)$$

$$V_{sq} = \left(\left(\frac{x_1 - x_2}{2}\right)^2, \cdot\right)$$

is symmetric

$$\text{so } x_{1,2} = \frac{x_1 + x_2}{2} \pm \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2}$$

$$x^5 + a_1 x^4 + \dots + a_5 = 0$$

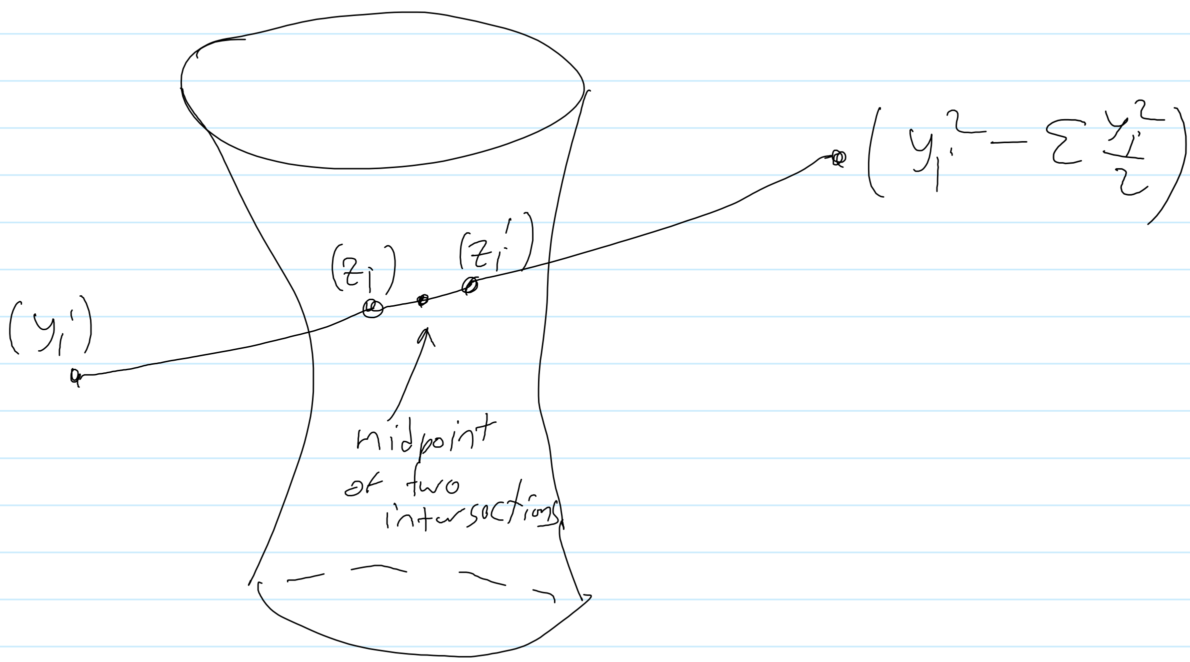
cannot be solved in radicals.

$$x_i \longmapsto x_i - \frac{\sum x_i}{5} = y_i$$

$$y^5 + 0y^4 + \dots = 0$$

Try to reach $\sum z_i z_j = 0$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_5 \end{pmatrix} \rightsquigarrow \begin{pmatrix} y_1^2 - \sum y_i^2/5 \\ \vdots \\ y_5^2 - \sum y_i^2/5 \end{pmatrix}$$



$\rightsquigarrow$ at the cost of a square root, get to the eq'n

$$z^5 + 0z^4 + 0z^3 + \cdots = 0$$

Now rescale the z's to get a quadratic surface in projective space.