β calculus

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9:56 AM

Tail Actions.

$$tm: \begin{array}{c|c} x & \alpha \\ y & \beta \\ & \gamma \end{array} \longmapsto \begin{array}{c|c} z & \alpha+\beta \\ & \gamma \end{array}, \quad C_x, C_y \to C_z$$

$$t\Delta: \begin{array}{c|c} z & \xi \\ & \gamma \end{array} \longmapsto \begin{array}{c|c} x & \xi \\ y & \xi \\ & \gamma \end{array}, \quad C_z \to C_x + C_y$$

$$tS: \begin{array}{c|c} z & \xi \\ & \gamma \end{array} \longmapsto \begin{pmatrix} -\xi \\ \gamma \end{pmatrix} \quad C_z \mapsto -C_z$$

Head Actions.

$$hm: \begin{array}{ccc} \left(\alpha \right. & \beta & \left| \gamma \right) \\ x & y & \end{array} \longmapsto \begin{array}{cc} \left(\alpha+\beta+(C\cdot\alpha)\beta \right. & \left| \gamma \right) \\ z & \end{array}$$

$$h\Delta: \begin{array}{cc} (\alpha & |\gamma) \\ z & \end{array} \longmapsto \begin{array}{ccc} (\alpha & |\alpha & |\gamma) \\ x & y & \end{array}$$

$$hS: (\alpha | \gamma) \longmapsto \left(\frac{-\alpha}{1+C\cdot\alpha} \middle| \gamma \right)$$

Factorization.

$$\begin{array}{c} x\text{tails} \\ y\text{tails} \\ \\ \end{array} \begin{array}{c} \begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix} \\ z \quad\quad \end{array} \longmapsto \begin{array}{c} \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \beta/_{1+C_x\cdot\alpha} & \delta \end{pmatrix} \\ x \quad y \quad\quad\quad \end{array}$$

Conjugation

$$y \begin{pmatrix} \alpha & \beta \\ \ldots \end{pmatrix} \longmapsto \begin{pmatrix} \alpha\left(1+\frac{C_r\cdot\gamma}{1+C_y\cdot\alpha}\right) & \beta\left(1+\frac{C_r\cdot\gamma}{1+C_y\alpha}\right) \\ \ldots \end{pmatrix}$$

$$\begin{array}{cc} y & \\ r & \end{array}\!\!\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \atop \quad x \longmapsto \begin{pmatrix} \alpha\left(1+\frac{\cdots}{1+C_y\cdot\alpha}\right) & \beta\left(1+\frac{\cdots}{1+C_y\alpha}\right) \\ \frac{\gamma}{1+C_y\cdot\alpha} & \frac{\delta-C_y(\alpha\delta+\gamma\cdot\beta)}{1+C_y\alpha} \end{pmatrix}$$

$$=\begin{pmatrix} \alpha & \beta \\ 0 & \delta \end{pmatrix}+\frac{1}{1+C_y\cdot\alpha}\begin{pmatrix} (C_r\cdot\gamma)\alpha & (C_r\cdot\gamma)\beta \\ \gamma & -C_y\gamma\cdot\beta \end{pmatrix}$$

while $W \longmapsto W_{new}=W\cdot(1+C_y\alpha)$

(so if $\alpha=\alpha'/W$, then $W\mapsto W_{new}=W\cdot\left(1+C_y\frac{\alpha'}{W}\right)$

$$\simeq W+C_y\alpha'$$

The R-Matrix.

$$R_{xy}=x\left(\frac{e^{C_x}-1}{C_x}\right)_{y} \qquad R^{-1}_{xy}=x\left(\frac{e^{-C_x}-1}{C_x}\right)_{y}$$

$$\frac{e^{C}-1}{C}+\frac{e^{-C}-1}{C}+\frac{e^{-C}-1}{C}\cdot\left(\frac{e^{C}-1}{C}\cdot C\right)=$$

$$=\frac{e^{C}-1}{C}+\frac{e^{-C}-1}{C}e^{C}=\frac{e^{C}-1+1-e^{C}}{C}=0$$