β braiding

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9:25 AM

(Following betaBraiding.nb)

```
{β1 = W[1] + Sum[α_{10 i+j} ar[i, j], {i, 3}, {j, 3}],
   β1 ** ar[1, 2],
   β1 ** ((-1)/(1 + c[1]) ar[1, 2])
 } /. c[i_] :→ c_i // βForm // ColumnForm
```

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12}+\frac{(1+c_1\alpha_{11})(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31}} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22}+\frac{c_1\alpha_{21}(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31}} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32}+\frac{c_1\alpha_{31}(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31}} & \alpha_{33} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12}-\frac{(1+c_1\alpha_{11})(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{(1+c_1)(1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31})} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22}-\frac{c_1\alpha_{21}(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{(1+c_1)(1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31})} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32}-\frac{c_1\alpha_{31}(1+c_1\alpha_{12}+c_2\alpha_{22}+c_3\alpha_{32})}{(1+c_1)(1+c_1\alpha_{11}+c_2\alpha_{21}+c_3\alpha_{31})} & \alpha_{33} \end{pmatrix}$$

Claim. Let $R=\mathbb{Z}[c_1 .. c_n]$ then PwB_n acts (non-linearly) on $M_{n\times n}(R)$ via

$$(v_1 .. v_i \dots v_j \dots v_n) \xmapsto{\sigma_{ij}} (\dots v_i \dots v_j' \dots)$$

where $v_j' = v_j + \frac{1+\gamma\cdot v_j}{1+\gamma\cdot v_i}(c_i v_i + e_i)$

where e_i is the standard unit vector

& $\gamma = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$.