0. Feynman’s 8 things.

1. Prelim 1: Poincare’s lemma: $dW = 0 \Rightarrow \exists \eta \text{ s.t. } d\eta = W$

2. Prelim 2: Integration by parts: $\int dW \wedge \eta = -\int W \wedge d\eta$

3. The Hodge *: $W^*(\eta) = \left< W/\eta \right> dx^1 \wedge \ldots \wedge dx^n$

Example, on $R^3_{xyz}$: $W^*(d^2t) = 0$

* $d(dx \wedge dy) = -dxdy$
* $d(dy \wedge dz) = -z^2 dt$
* $d(z^2 dx) = -dx^2 dt$

4. The first action principle to $\mathbb{R}^3$.

5. $A \in \mathcal{C}_0^1(\mathbb{R}^3_{xyz}) \quad J \in \mathcal{C}_0^2(\mathbb{R}^3)$

$S(A) = \int_{\mathbb{R}^3} \left[ -\frac{1}{2} \text{curl} A \wedge \text{curl} A + J \wedge A \right] dV$

$\text{turn part to } \phi$

in $S(A+eB) = 0 = \int_{\mathbb{R}^3} \left[ \left( B \wedge \text{curl} A + dA \wedge dB + J \wedge B \right) \wedge dA \right.$

$\left. + J \wedge A \right]$

$= \int_{\mathbb{R}^3} B \wedge (d\wedge dA) - B \wedge J = \int_{\mathbb{R}^3} B \wedge (d\wedge dA - J)$

$\Rightarrow \quad d \wedge dA = J$

6. Therefore $dJ = 0$, $\text{curl} F = 0 \quad dF = 0 \quad d \wedge F = J$.

7. Write $F = E \wedge dx \wedge dt + \ldots \quad J = \rho \wedge dx \wedge dt$

$+ B \wedge dy \wedge dt + \ldots \quad - \omega \wedge dy \wedge dt + \ldots$

$\text{d}J = 0 : \left( \frac{\partial \rho}{\partial t} + \frac{\partial \omega}{\partial x} + \ldots \right) dtdx \wedge dy \wedge dz$

so $\frac{\partial \rho}{\partial t} + \text{div} \, J = 0$

“conservation of charge”

[BTW, charge current really]
\[ \oint F = 0 \quad \text{curl of } \nabla \cdot \mathbf{D} = 0 \]
\[ \oint \mathbf{D} \cdot d\mathbf{S} = 0 \quad \rho = \frac{1}{\varepsilon_0} \int \mathbf{E} \cdot d\mathbf{S} \]
\[ \oint \mathbf{B} \cdot d\mathbf{S} = 0 \quad \mu_0 \int \mathbf{H} \cdot d\mathbf{S} = 0 \]
\[ \nabla \cdot \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \]

Physical interpretation

8. The ellipticity problem & the Lorentz force.