# Inverting the Laplacian

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$$\Delta = d^* d$$

Claim. on $\mathbb{R}^n$, $\Delta^{-1}(x,y) = \frac{1}{|x-y|^{n-2}} =: K(x,y)$

Pf $\langle f, \Delta K g\rangle = \langle df, dKg\rangle$

$$\widetilde{\frac{1}{|p|^2}} =$$

$$\int dp_0\, dp' \frac{1}{p_0^2 + |p'|^2} \cdot e^{-ixp_0} =$$

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Working proof:

1. Compute $\Delta \frac{1}{|x|^{n-2}}$ away from $0$ by hand.
2. Compute $\operatorname{div}\operatorname{grad} \frac{1}{|x|^{n-2}}$ at $0$.