Facts and Dreams About $v$-Knots and Etingof–Kazhdan, I

Abstract. I will describe, to the best of my understanding, the relationship between virtual knots and the Etingof-Kazhdan quantization of Lie bialgebras, and explain why, IMHO, both knot theorists and algebraists should care. I am not happy yet about the state of my understanding of the subject but I haven’t lost hope of achieving happiness, one day.

Abstract Generalities. $(K, I)$: an algebra and an “augmentation ideal” in it. $K := \lim_{n \to \infty} K/I^n$ the “$I$-adic completion”. $gr_I K := \bigoplus_{n \geq 0} I^n/I^{n+1}$ has a product $\mu$, specifically, $\mu_{11} = (1/I^2)\mu_2 \to I^2/I^3$. The “quadratic approximation” $A_2(K) := FC(\ker \mu_{11})$ of $K$ subjects using $\mu$ on $gr K$.

The Prized Object. A “homomorphic $A$-expansion”: a homomorphic filtered $\mathcal{Z} : K \to A$ inducing the identity on $I/I^2 = C$.

Dror’s Dream. All interesting graded objects and equations, especially those around quantum groups, arise this way.

Example 1. $K = \mathbb{Q} Rep_n$ “braids when you look”, [Lee] shows that a non-homomorphic $\mathcal{Z}$ exists. [BEER]: there is no homomorphic one.

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General Algebraic Structures.

- Has kinds, elements, operations, and maybe constants. All still works.
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Example 3. Quandle: a set $Q$ with an op $\wedge$ s.t.

$1 \wedge x = 1, x \wedge 1 = x = x \wedge x$.

Expected properties:

1. $x \wedge (y \wedge z) = (x \wedge y) \wedge z$.

Example 4. Parenthesized braids make a category with some extra operations. An expansion is the same thing as an associator, and the Grothendieck-Teichmüller story arises naturally.
To Do:

- Example: KTGs and non-horizontal associators. ("bracket rise" arises here).
- Example: wKO's and the Kashiwara-Vergne equations.
- vKO's, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
- wKO's, uKO's, and Alekseev-Enriquez-Torresian.

Footnotes and references are on the PDF version, page 3.
Footnotes

1. I probably mean “a functor from some fixed “structure multi-category” to the multi-category of sets, extended to formal linear combinations”.

2. See my paper [BN1] and my talk/handout/video [BN2].

References


