Facts and Dreams About v-Knots and Etingof-Kazhdan, 1

Abstract. I will describe, to the best of my understanding, the relationship between virtual knots and the Etingof-Kazhdan quantization of Lie bialgebras, and explain why, in my humble opinion, both topologists and algebraists should care. I am not happy yet about the state of my understanding of the subject but I haven’t lost hope of achieving happiness, one day.

Abstract Generalties

(R, I): an algebra and an “augmented” in it.

\[ C^\text{cl} \cong \frac{R}{I} \text{ “The Etingof-quantum”} \]

\[ Y \ast R := \bigoplus_{n \geq 1} \frac{R}{I^n} \quad \text{has a product} \]

\[ A^\text{h}(K) := TV(V = \mathbb{R}) \text{ a “homotopy” approximation} \]

of \( K \), surjective by \( \mu \) on \( Y \ast R \).

The Pifted Object: A “homomorphic” expansion

\[ \pi^\text{h} : R \to A^\text{h}(K) \text{ which induces} \]

homomorphic the identity on \( \mathbb{R} \to V \).

Dror’s Dream. All interesting graded objects and equations in mathematics, especially around quantum groups, arise this way.

To Do.

- The Peter Lee setup for \( R, I \), “all interesting graded equations arise in this way”.
- Example: the pure braid group (mention \( PEB \), too).
- Generalized algebraic structures.
- Example: quandles.
- Example: parenthesized braids and horizontal associators.
- Example: KTGM and non-horizontal associators. (“Bracket rise” arises here).
- Example: wKO’s and the Kashiwara-Vergne equations.
- vKO’s, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
- wKO’s, uKO’s, and Alekseev-Enriquez-Torrosian.
- The third page.