Operads

April-23-11
11:38 AM

From http://en.wikipedia.org/wiki/Operad:

Definition

[edit]

In category theory, an **operad without permutations** (sometimes called a **non-symmetric**, **non-Σ** or **plain** operad) is a multicategory with one object. More explicitly, such an operad consists of the following:

This is a key weakness.

- a sequence $(P(n))_{n\in\mathbb{N}}$ of sets, whose elements are called *n-ary operations*,
- for each integers n, k_1, ..., k_n a function

  $$\begin{array}{ccc} P(n)\times P(k_1)\times\cdots\times P(k_n) & \to & P(k_1+\cdots+k_n) \\ (\theta,\theta_1,\ldots,\theta_n) & \mapsto & \theta\circ(\theta_1,\ldots,\theta_n) \end{array}$$

  called *composition*,
- an element 1 in $P(1)$ called the *identity*,

satisfying the following coherence properties:

- *associativity*:

  $$\theta\circ(\theta_1\circ(\theta_{1,1},\ldots,\theta_{1,k_1}),\ldots,\theta_n\circ(\theta_{n,1},\ldots,\theta_{n,k_n}))=(\theta\circ(\theta_1,\ldots,\theta_n))\circ(\theta_{1,1},\ldots,\theta_{1,k_1},\ldots,\theta_{n,1},\ldots,\theta_{n,k_n})$$
- *identity*:

  $$\theta\circ(1,\ldots,1)=\theta=1\circ\theta$$

(where the number of arguments correspond to the arities of the operations).

A **morphism of operads** $f:P\to Q$ consists of a sequence

$$(f_n:P(n)\to Q(n))_{n\in\mathbb{N}}$$

which:

- preserves composition: for every n-ary operation θ and operations θ_1, ..., θ_n,

  $$f(\theta\circ(\theta_1,\ldots,\theta_n))=f(\theta)\circ(f(\theta_1),\ldots,f(\theta_n))$$
- preserves identity:

  $f(1)=1$.

Operads were originally defined topologically, by May, but his full definition requires symmetric group actions on the $P(n)$ that are suitably related to the maps θ_n. The permutation actions are additional structure that is vital to the original and most later applications.