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Classical Dynamics, $\phi^t : X \to X$, Billiards, Geodesic Flow,
Ergodicity [time average converge to space averages].

Quantum System. $U^t : \mathcal{H} \to \mathcal{H}$; e.g. $\mathcal{H} = L^2(M)$

$$U^t = \exp\left(\frac{i\hbar t}{2m}\Delta\right), \text{ or, w/ } \psi_t = U^t \psi,$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi$$

Can be lifted to W_ψ on $X = TM$, [W of Wigner, "Wigner Dist."]
with

$$W_{\psi_t}(f) = W_{\psi_0}(f \circ \phi^t) + O_t(\hbar)$$

Stationary states:

$$U^t \psi = e^{iEt}\psi, \quad \Delta\psi = \lambda\psi, \quad \lambda = \frac{2mE}{\hbar}$$

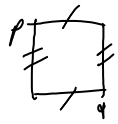
We expect that high E solutions will have nearly-uniform W_ψ's.

Theorem. If φ_j is a basis of $L^2(M)$ w/ $\Delta\varphi_j = \lambda_j \varphi_j$. Then

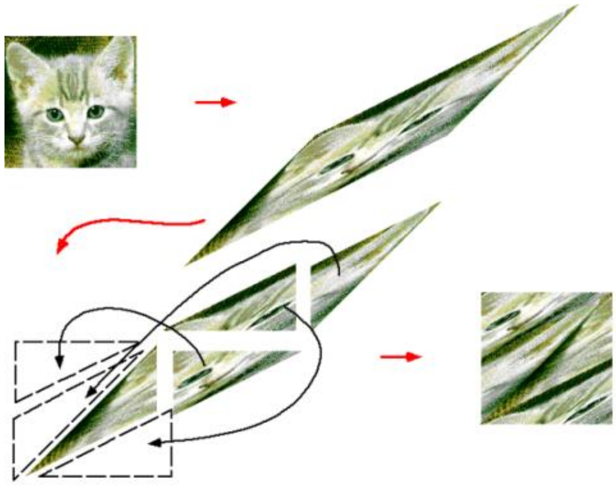
$$\frac{1}{N(T)} \sum_{T \le \lambda_j \le 2T} \left| W_{\varphi_j}(f) - \int f \right|^2 \xrightarrow[T \to \infty]{} 0$$

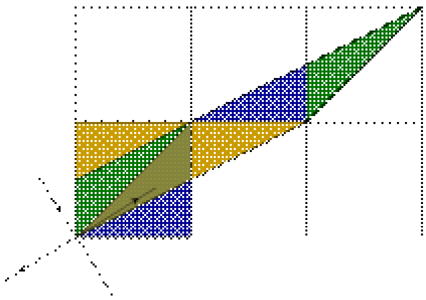
$\uparrow$ # of eigenvalues in range.

Quantized Cat Map.

$$X = \mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2 =$$ 

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \in SL_2(\mathbb{Z}), \quad \begin{pmatrix} p \\ q \end{pmatrix} \mapsto A\begin{pmatrix} p \\ q \end{pmatrix}$$





Picture showing how the linear map stretches the unit square and how its pieces are rearranged when the modulo operation is performed. The lines with the arrows show the direction of the contracting and expanding eigenspaces

There is an N-dimensional quantization... 11:44