Arkhipov on Schemes, January 11

January-11-11
11:09 AM

Ravi Vakil - Lecture notes on the web.

Textbook: "Schemes & arithmetic surfaces"

<u>Definition</u> A pre-sheaf on a topological space X is a contra-variant functor from the category $C(X)$ of open sets in X to (Sets or Ab or Com (com. rings), etc).

<u>Example</u> Given $x \in X$, the skyscraper pre-sheaf is

$$\mathcal{F}_x(U) = \begin{cases} \mathbb{Z} & x \in U \\ 0 & x \notin U \end{cases}$$

<u>Example</u> The constant pre-sheaf.

<u>Example</u> Functions & restriction maps.

<u>Def</u> A morphism between two functors $F, G : C_1 \to C_2$.

[Functors $C_1 \to C_2$ form a category]

Given $C_1 \underset{R}{\overset{L}{\rightleftarrows}} C_2$, L is left adjoint to R & R is right adjoint to L if $\forall \begin{smallmatrix} X \in Ob(C_2) \\ Y \in Ob(C_1) \end{smallmatrix}$

we have a canonical isomorphism

$$Hom_{C_1}(L(X), Y) \xrightarrow{\sim} Hom_{C_2}(X, R(Y))$$

Such that

$$id \in Hom(L(X), L(X)) \longrightarrow \epsilon_X \in Hom(X, RL(X))$$

$$id \in Hom(R(Y), R(Y)) \longrightarrow \delta_Y \in Hom(LR(Y), Y)$$

$$F := RL \qquad G := LR$$

$$F \circ F = LRLR = LGR \to L \cdot I \cdot R = LR = F$$

$$G \longrightarrow G \circ G$$

The axiom is 1. F is a monad.

2. G is a co-monad.

<u>Examples</u> 1. $C_1 = \text{Vect}$ $\quad C_2 = \text{set}$

$R(V) = V$ as a set

$L(S) = \langle S \rangle$

2. $C_1 = \text{Grps}$ $\quad C_2 = \text{Ab}$

$\text{Ab} \xrightarrow{R} \text{Grp}$ $\qquad \text{Grp} \xrightarrow{L} \text{Ab}$ $\qquad L(G) = G/[G,G]$

3. $C_1 = \text{Rings}$ $\quad C_2 = \text{Ab}$

$R : \text{Rings} \to \text{Ab}$ $\qquad L : A \mapsto \text{Free}_{\mathbb{Z}}(A)$