

t_n L_∞ -alg ICG

$g = \sum_{i=1}^n g[i] \hookrightarrow d_{CE}$
comm. rules

x_1, \dots, x_n

$\sum_{\sigma \in S_n} \sum_{\text{shuffles}} (x_{\sigma(1)} \dots x_{\sigma(n)}) \otimes (x_{\sigma(1)} \dots x_{\sigma(n)})$

$H(g, id)$ is Lie alg

$\sum_{\alpha \in n} \mu_\alpha(x_1, \dots, x_n) = 0$

Def L_∞ -alg str on $g \Leftrightarrow \text{diff Dom } (D^2 g)$

$\sum g[i]$

\downarrow
 $g[i]$

(=) Set of maps

$M_n: S^n g[1] \rightarrow g[1]$
 M_2 : "Lie bracket"
 M_n : diff. id

$[x, y, z] + \mathbb{Q} = dM_3(x, y, z) - \mu_3(dx, y, z)$

Has anything good
we can come out of
 ∞ -objects?

t_n ICG L_∞ -algebra, $H(ICG) = t_n$

to define

Graphs(n) = { Lin. comb of graphs (undir) with
n "external" vert (numbered)
 ≥ 0 "internal" vert }

signs & degrees
edge degrees ≥ 1
no tadpoles

Wants ≥ 3 -valent

connect edges in all possible ways

Diff: $\delta \Pi = \sum_{\text{vert}} (\text{splitting at v})$

t_n ICG L_∞ -algebra, $H(ICG) = t_n$
 $H(\text{Graphs}(n)) = e_2$ Gerstenhaber operad
 Kontsevich, Lambrechts, Valeri
 e_N -algebra structure on V
 (\Leftrightarrow)

- \cdot, \wedge gr comm product on V
- $[\cdot, \cdot]$ makes $V[N-1]$ into a Lie alg
- $[x, ynz] = [x, y]nz \pm \tilde{y}n[x, z]$

 FM_N : space of conf. of n points in \mathbb{R}^N
 $H_0(FM_N(n)) = e_N(n)$

$(N=2)$ ICG L_∞ -algebra, $H(ICG) = t_n$
 $C_0(FM_2) \rightarrow \text{Graphs}_2$
 $c \mapsto \sum \Gamma \cdot \left(\int_c \omega_\Gamma \right)$
 $\text{Graphs}(n) = C_0 \oplus C_1 \oplus C_2 \oplus \dots$
 Spectral seq: $E_1 = C_0 \oplus C_1 \oplus C_2 \oplus \dots$
 $\int \omega_\Gamma = \frac{d \text{Arg}(z_i - z_j)}{2\pi}$
 $\frac{d\varphi}{2\pi}$
 $e_2 \rightarrow \text{Graphs}$
 closed graphs w/o external vertices

$(N=2)$ ICG L_∞ -algebra, $H(ICG) = t_n$
 $\text{Graphs} = S(ICG[1])$
 int connected graphs
 δ compat. with coproduct
 \Rightarrow ICG(n) is an L_∞ -algebra
 $M_2(\Gamma_1, \dots, \Gamma_k)$: glue at externals
 \rightarrow split so that only one int. conn. comp. remains
 $H(C(ICG)) = e_2 = H(CCE)$ destruct $H(ICG) = t$

$t_n = t_{n-1} \times \text{FreeLie}(n-1)$

$\text{TCG} \rightarrow \text{ICG}(n) = \text{ICG}(n-1) \oplus \text{ICG}(n)'$

$\exists \text{ edges connecting to } n$

$\text{ICG} \rightarrow \text{ICG}' \rightarrow \text{tder}$

$C_1 = \text{graphs}$

filtration by nr of conn comp.

$\rightarrow \text{obtain}$

$S^{22}(\text{FreeLie}(n-1)[1])_{\text{tder}}$

$S^{22} / \text{tder} \rightarrow S^c = \text{FreeLie}(n-1)$