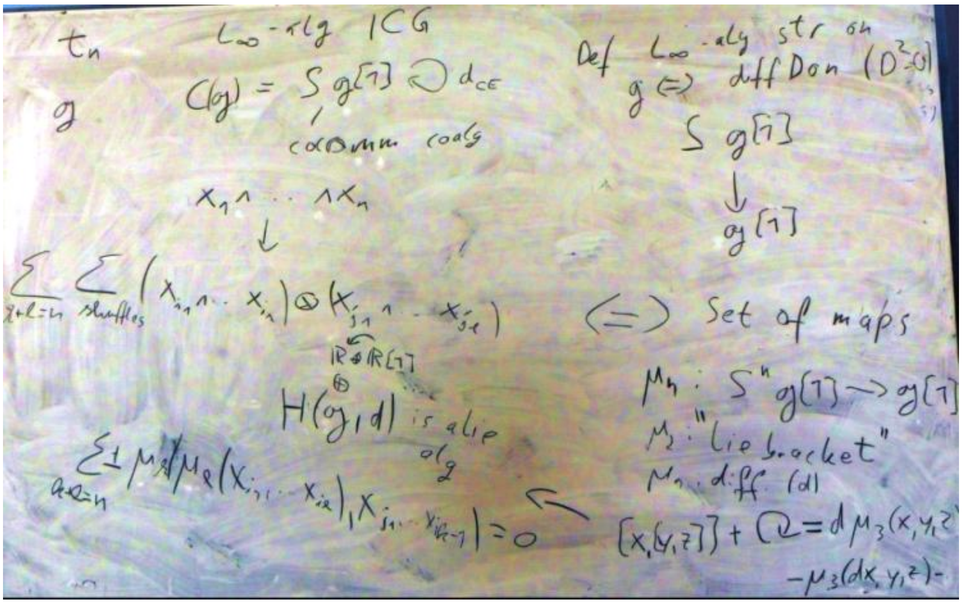
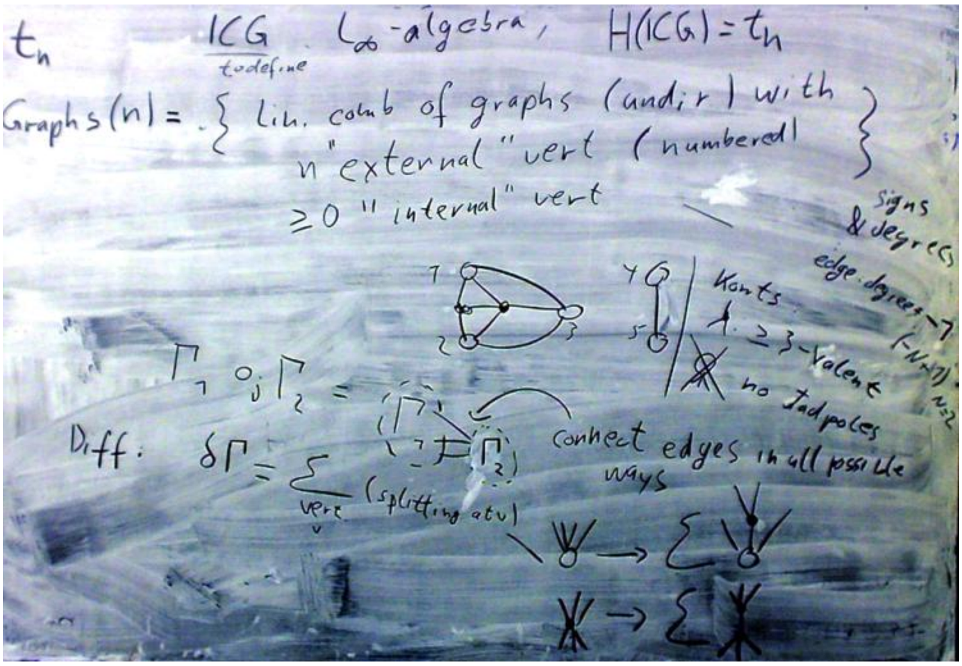
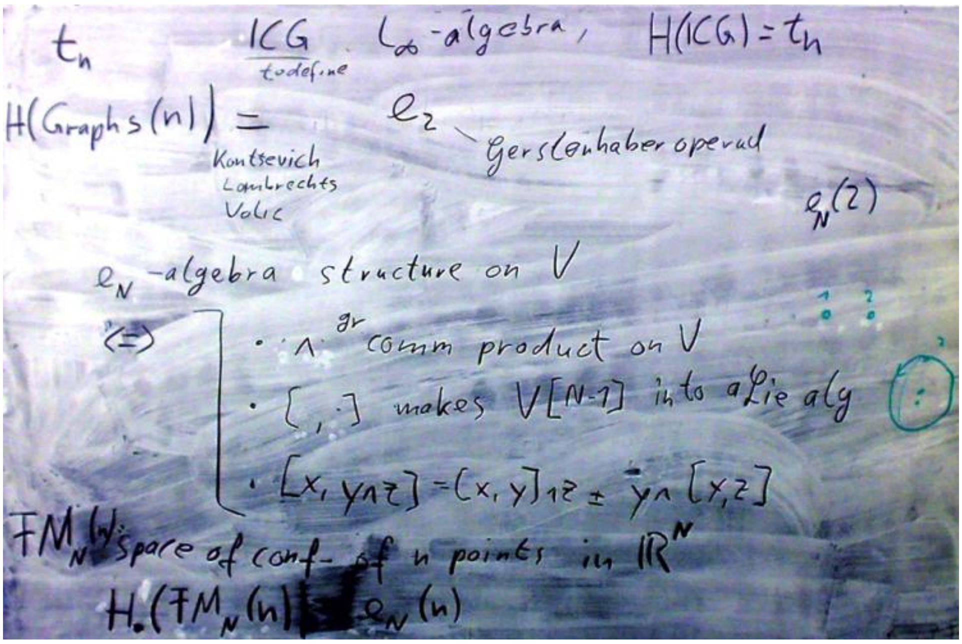
Willwacher, Dec 1

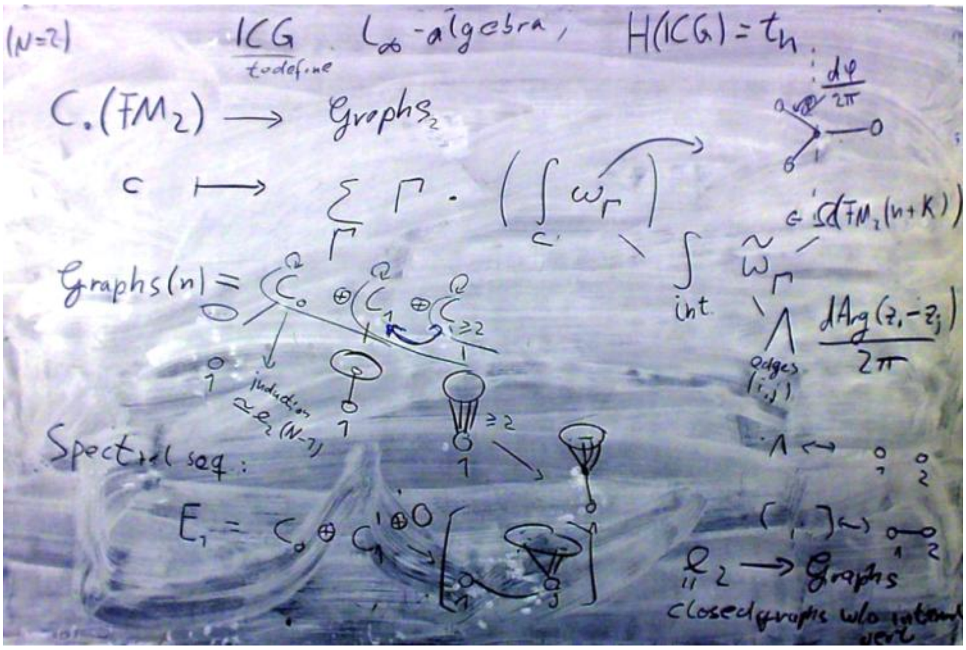
December-12-10
11:57 AM

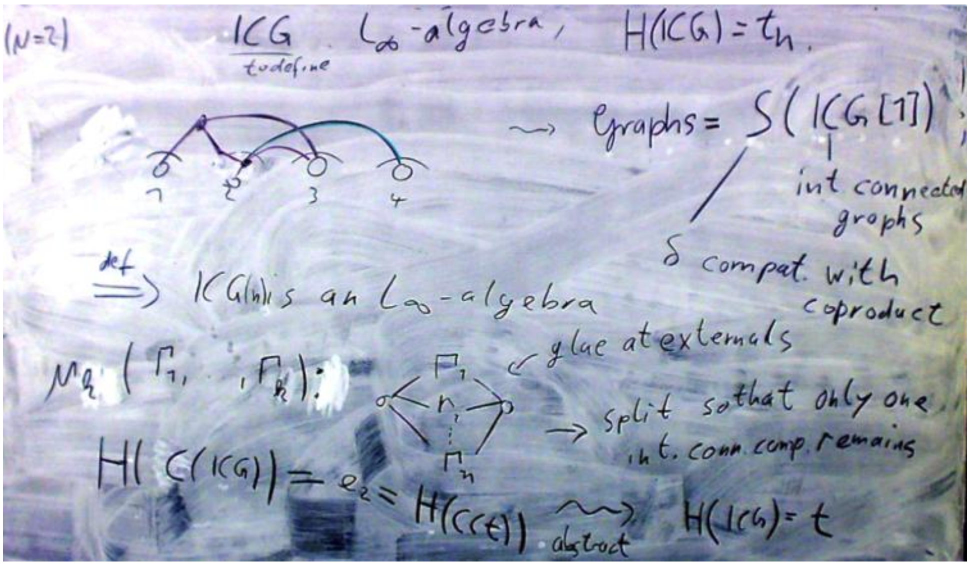




Has anything good ever come out of ∞-objects?


t_n
ICG
to define
L_∞-algebra, H(ICG) = t_n
H(Graphs(n)) = e_2
Kontsevich
Lambrechts
Volic
Gerstenhaber operad
e_N(2)
e_N-algebra structure on V
⟺
· ∧ gr comm product on V
· [,] makes V[N-1] into a Lie alg
· [x, y∧z] = [x,y]∧z ± y∧[x,z]
FM_N(n): space of conf. of n points in ℝ^N
H_•(FM_N(n)) = e_N(n)


(N=2)
ICG
to define
L_∞-algebra, H(ICG) = t_n
C_•(FM_2) ⟶ Graphs_2
c ⟼ Σ_Γ Γ · (∫_c ω_Γ)
dφ/2π
∈ Ω(FM_2(n+K))
∫ int
ω̃_Γ
∧ edges (i,j) dArg(z_i − z_j)/2π
Graphs(n) = C_0 ⊕ C_1 ⊕ C_{≥2}
induction
≃ e_2(N-1)
≥2
Spectral seq:
E_1 = C_0 ⊕ C_1 ⊕ 0
∧ ↔
[,] ↔
e_2 ⟶ Graphs
closed graphs w/o internal vert


(N=2)
ICG
to define
L_∞-algebra, H(ICG) = t_n
1 2 3 4
⇝ Graphs = S(ICG[1])
int connected graphs
δ compat. with coproduct
def ⟹ ICG(n) is an L_∞-algebra
glue at externals
μ_k(Γ_1, ..., Γ_k):
→ split so that only one int. conn. comp. remains
H(C(ICG)) = e_2 = H(C(t)) ⇝ abstract H(ICG) = t

$TCG \nearrow t_n = t_{n-1} \ltimes \mathrm{FreeLie}(n-1)$

$\searrow ICG(n) = ICG(n-1) \oplus ICG(n)' \leftarrow \exists$ edges connecting to n

$ICG \to ICG^{or} \to \mathfrak{tder}$

$C_1 \leftrightarrow C_{\geq 2}$

$=1$ $\quad$ ≥ 2

$\rightsquigarrow C_1' =$ graphs

filtration by nr of conn comp.

ext

$\longrightarrow$ obtain

$S^{\geq 2} / d_{CE} \cdot S^3 \to S^2 \quad S^{\geq 2}(\mathrm{FreeLie}(n-1)[1]), d_{CE}$

$= \mathrm{FreeLie}(n-1)'$