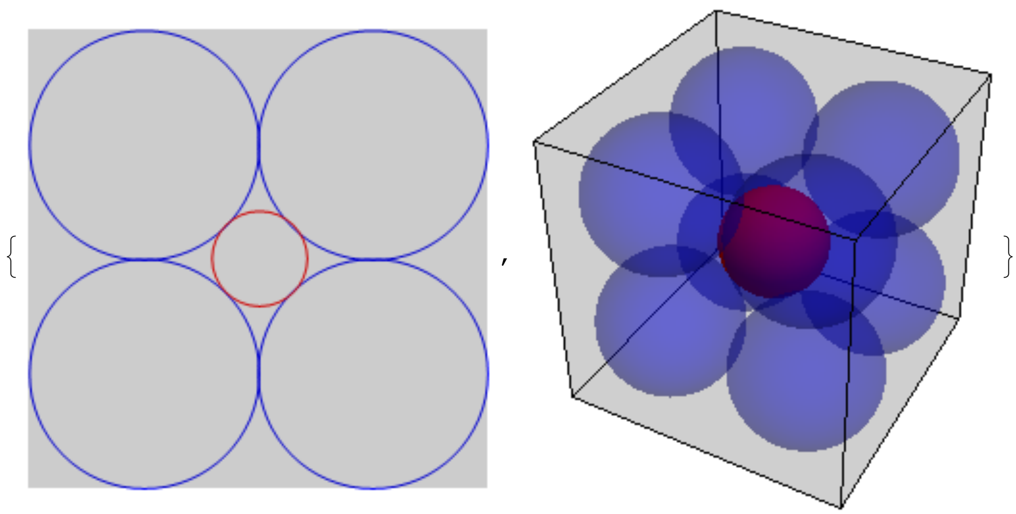
Pensieve Header: A riddle from MathOverflow, http://mathoverflow.net/questions/5372/dimension-leaps/5483#5483.

Put 2^n unit balls centered at the points (±1, ±1,..., ±1) in R^n; they are all bound in the cube C=[−2, 2]^n. Let B be the maximal ball centered at 0 and not intersecting with any of the previous balls. As n→∞, what's the limit of Vol(B)/Vol(C)?

```
Rasterize /@ {
  Graphics[{
    Blue, Circle[List@@#, 1] & /@ Flatten[Outer[Center, {-1, 1}, {-1, 1}]],
    Red, Circle[{0, 0}, Sqrt[2] - 1],
    Black, Opacity[0.2], Rectangle[{-2, -2}, {2, 2}]
   }, ImageSize → 250],
  Graphics3D[{
    Blue, Opacity[0.3],
    Sphere[List@@#, 1] & /@ Flatten[Outer[Center, {-1, 1}, {-1, 1}, {-1, 1}]],
    Red, Opacity[1], Sphere[{0, 0, 0}, Sqrt[3] - 1],
    Black, Opacity[0.1], Cuboid[{-2, -2, -2}, {2, 2, 2}]
   }, Boxed → False, ImageSize → 250]
 }
```



```
WolframAlpha["Volume of n-sphere", "Result"]
```

$$\frac{2\,\pi^{n/2}\,r^n}{n\,\Gamma\left(\frac{n}{2}\right)} \approx \frac{2\times 3.14159^{n/2}\,r^n}{n\,\Gamma\left(\frac{n}{2}\right)}$$

(assuming radius r)

$$\texttt{vr[n\_] :=} \frac{2\,\pi^{n/2}\left(\sqrt{n}-1\right)^n}{n\,\texttt{Gamma}\left[\frac{n}{2}\right]} \Big/ 4^n;\ \texttt{\{Table[vr[n], \{n, 2, 10\}], vr[100]\} // N}$$

```
{{0.0336883, 0.0256763, 0.0192766, 0.0148324, 0.0117012,
  0.00942996, 0.00773623, 0.0064424, 0.00543354}, 0.0000391445}
```