Put $2^n$ unit balls centered at the points $(\pm 1, \pm 1, \ldots, \pm 1)$ in $\mathbb{R}^n$; they are all bound in the cube $C=[-2, 2]^n$. Let $B$ be the maximal ball centered at 0 and not intersecting with any of the previous balls. As $n \to \infty$, what’s the limit of $\frac{\text{Vol}(B)}{\text{Vol}(C)}$?

```
WolframAlpha["Volume of n-sphere", "Result"]
```

\[
\frac{2 \pi^{n/2} r^n}{n \Gamma\left(\frac{n}{2}\right)} \approx \frac{2 \times 3.14159^{n/2} r^n}{n \Gamma\left(\frac{n}{2}\right)}
\]

(assuming radius $r$)

\[
vr[n_] := \frac{2 \pi^{n/2} \left(\sqrt{n} - 1\right)^n}{n \Gamma\left(\frac{n}{2}\right)} / 4^n; \{\text{Table}[vr[n], \{n, 2, 10\}], vr[100]\} // N
\]

\[
\{0.0336883, 0.0256763, 0.0192766, 0.0148324, 0.0117012, 0.00942996, 0.0073623, 0.0064424, 0.00543354, 0.000391445\}
\]