Pensieve Header: Testing TailScatteringTowardsTheHead.

In[1]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2010-09"];
<< KnotTheory`
<< TailScatteringTowardsTheHead-Program.m
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[4]:=
```
Union[
 Simplify[
    Det[Drop[R[GD[#]], None, -1]] / Alexander[#][X]
   ] & /@ AllKnots[{3, 10}]
]
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

Out[4]= $\left\{-1, 1, -\frac{1}{X^5}, -\frac{1}{X^4}, \frac{1}{X^4}, -\frac{1}{X^3}, \frac{1}{X^3}, -\frac{1}{X^2}, \frac{1}{X^2}, -\frac{1}{X}, \frac{1}{X}, -X, X, -X^2, X^2, -X^3, X^3, -X^4, X^4, -X^5, X^5, -X^6, X^6\right\}$

In[5]:=
```
Test[K_] := (K → Simplify /@ Expand /@ Equal[
       EZ[GD[K]] ,
       - x X D[Alexander[K][X], X] / Alexander[K][X] + (SL[K] /. DL → x + DR)
      ])
```

In[6]:=
```
Test /@ AllKnots[8]
```

Out[6]=
```
{Knot[8, 1] → True, Knot[8, 2] → True, Knot[8, 3] → True, Knot[8, 4] → True, Knot[8, 5] → True,
 Knot[8, 6] → True, Knot[8, 7] → True, Knot[8, 8] → True, Knot[8, 9] → True,
 Knot[8, 10] → True, Knot[8, 11] → True, Knot[8, 12] → True, Knot[8, 13] → True,
 Knot[8, 14] → True, Knot[8, 15] → True, Knot[8, 16] → True, Knot[8, 17] → True,
 Knot[8, 18] → True, Knot[8, 19] → True, Knot[8, 20] → True, Knot[8, 21] → True}
```

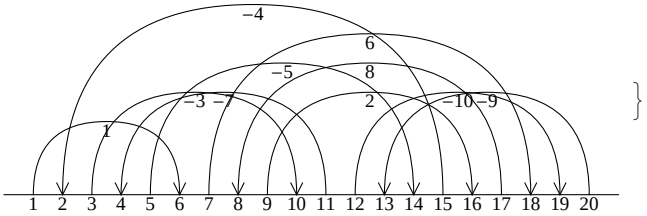
In[7]:=
```
Test2[K_] := (K → Simplify /@ Expand /@ Equal[
       EZ[GD[K]] /. DR → 0,
       EZ2[GD[K]]
      ])
```

In[8]:=
```
Test2 /@ AllKnots[8]
```

Out[8]=
```
{Knot[8, 1] → True, Knot[8, 2] → True, Knot[8, 3] → True, Knot[8, 4] → True, Knot[8, 5] → True,
 Knot[8, 6] → True, Knot[8, 7] → True, Knot[8, 8] → True, Knot[8, 9] → True,
 Knot[8, 10] → True, Knot[8, 11] → True, Knot[8, 12] → True, Knot[8, 13] → True,
 Knot[8, 14] → True, Knot[8, 15] → True, Knot[8, 16] → True, Knot[8, 17] → True,
 Knot[8, 18] → True, Knot[8, 19] → True, Knot[8, 20] → True, Knot[8, 21] → True}
```

In[9]:=
```
{K = Knot[10, 162], A = Alexander[K][X], gd = GD[K], n = Length[gd], arcs = Arcs[gd], Draw[gd]}
```

Out[9]=
$$\left\{\text{Knot}[10, 162], -11 - \frac{3}{X^2} + \frac{9}{X} + 9\,X - 3\,X^2,\right.$$
```
 GD[Ar[1, 6, 1], Ar[9, 16, 1], Ar[3, 10, -1], Ar[15, 2, -1], Ar[5, 14, -1],
  Ar[7, 18, 1], Ar[11, 4, -1], Ar[17, 8, 1], Ar[20, 13, -1], Ar[12, 19, -1]],
 10, {1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 6, 7, 8, 8, 9, 9, 10, 11},
```



In[10]:=
```
ez2 = EZ2[gd]
```

Out[10]=
$$\frac{x\,X\left(9 - 22\,X + 27\,X^2 - 12\,X^3\right)}{3 - 9\,X + 11\,X^2 - 9\,X^3 + 3\,X^4}$$

In[11]:=
```
Simplify[Expand[Plus @@ EZ1[gd] /. DR → 0]]
```

Out[11]=
$$\frac{x\,X\left(9 - 22\,X + 27\,X^2 - 12\,X^3\right)}{3 - 9\,X + 11\,X^2 - 9\,X^3 + 3\,X^4}$$

In[12]:=
```
R[gd] // MatrixForm
```

Out[12]//MatrixForm=
$$\begin{pmatrix}
1-X & 0 & -1 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 & 0 \\
0 & 1-\frac{1}{x} & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 & 0 & 0 & 0 \\
-1 & \frac{1}{x} & 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 \\
0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 & 0 \\
0 & 0 & 0 & 1-X & 0 & 0 & 0 & 0 & -1 & X & 0 \\
0 & -1 & \frac{1}{x} & 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & X & 0 & 0 & 0 & 1-X & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 & 0 & 1-\frac{1}{x} \\
0 & 0 & 0 & 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & -1 & \frac{1}{x}
\end{pmatrix}$$

In[13]:=
```
MatrixForm /@ {M = Drop[R[gd], None, -1], DM = X D[M, X]}
```

Out[13]=
$$\left\{\begin{pmatrix} 1-X & 0 & -1 & X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 \\ 0 & 1-\frac{1}{x} & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 & 0 & 0 \\ -1 & \frac{1}{x} & 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{x} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 \\ 0 & 0 & 0 & 1-X & 0 & 0 & 0 & 0 & -1 & X \\ 0 & -1 & \frac{1}{x} & 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & X & 0 & 0 & 0 & 1-X & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{x} & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} -X & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X & 0 & 0 & 0 & X & 0 \\ 0 & \frac{1}{x} & 0 & 0 & 0 & -\frac{1}{x} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{x} & 0 & 0 & 0 & 0 & 0 & \frac{1}{x} & 0 & 0 \\ 0 & 0 & \frac{1}{x} & 0 & 0 & 0 & 0 & -\frac{1}{x} & 0 & 0 \\ 0 & 0 & 0 & -X & 0 & 0 & 0 & 0 & 0 & X \\ 0 & 0 & -\frac{1}{x} & 0 & 0 & \frac{1}{x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & -X & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{x} & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

In[14]:=
```
EA = Simplify[Expand[-x Tr[Inverse[M].DM]]]
```

Out[14]=
$$\frac{3x\left(2-3X+3X^3-2X^4\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[15]:=
```
IM = Append[Inverse[Drop[R[gd], None, -1]], Table[0, {n}]];
j = 0;
Simplify[Expand[-x (Plus @@ (gd /. Ar[t_, h_, s_] :>
        (++j; (-IM[[arcs[[t]], j]] + IM[[arcs[[h]] + 1, j]]) s X^s)
    ))]]
```

Out[17]=
$$\frac{3x\left(2-3X+3X^3-2X^4\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[43]:=
```
IM = Inverse[Append[R[gd], e[n + 1, n + 1]]];
j = 0;
Simplify[Expand[-x (Plus @@ (gd /. Ar[t_, h_, s_] :>
        (++j; (-IM[[arcs[[t]], j]] + IM[[arcs[[h]] + 1, j]]) s X^s)
    ))]]
```

Out[45]=
$$\frac{3x\left(2-3X+3X^3-2X^4\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[18]:=
```
Simplify[EA - ez2]
```

Out[18]=
$$2x$$

In[19]:=
```
j = 0;
Simplify[Expand[-x (List @@ (gd /. Ar[t_, h_, s_] :>
        (++j; (-IM[[arcs[[t]], j]] + IM[[arcs[[h]] + 1, j]]) s X^s)
    ))]]
```

Out[20]=
$$\left\{\frac{x\left(-1+2X+X^2-3X^3+4X^4-2X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{x\left(-1+2X+X^2-3X^3+3X^4-X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\right.$$
$$\frac{x\left(-1+6X-11X^2+7X^3-2X^4\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ -\frac{x\left(1-4X+5X^2-3X^3+X^4\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},$$
$$\frac{x\left(2-5X+4X^2-2X^3\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{x\left(-1+6X^2-9X^3+8X^4-3X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{2x(-1+X)^2\left(2-2X+X^2\right)}{3-9X+11X^2-9X^3+3X^4},$$
$$\left.\frac{x\left(1-3X+5X^2-5X^3+4X^4-2X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ -\frac{x(-1+X)^3}{3-9X+11X^2-9X^3+3X^4},\ \frac{x\left(2-5X+5X^2-5X^3+2X^4\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)}\right\}$$

In[21]:=
```
Simplify[Expand[EZ1[gd] /. DR → 0]]
```

Out[21]=
$$\left\{\frac{x\left(-1+5X-8X^2+8X^3-5X^4+X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{x\left(-1+5X-8X^2+8X^3-6X^4+2X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\right.$$
$$-\frac{x(-1+X)^2\left(1-X-X^2+3X^3\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{x\left(-1+X+4X^2-8X^3+8X^4-3X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ -\frac{x(-1+X)^2\left(-2+4X-3X^2+3X^3\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},$$
$$-\frac{x\left(1-3X+3X^2-2X^3+X^4\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ \frac{x\left(1-3X+3X^2+X^3-X^4\right)}{3-9X+11X^2-9X^3+3X^4},\ \frac{x\left(1-4X^2+6X^3-5X^4+X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},$$
$$\left.\frac{x\left(-2+6X-8X^2+8X^3-3X^4\right)}{3-9X+11X^2-9X^3+3X^4},\ \frac{x\left(2-8X+14X^2-16X^3+11X^4-3X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)}\right\}$$

In[22]:=
```
Det[Drop[R[gd], None, {#}]] & /@ Range[Length[gd]]
```

Out[22]=
$$\left\{11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2,\ -11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2,\ 11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2,\right.$$
$$-11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2,\ 11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2,\ -11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2,$$
$$\left.11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2,\ -11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2,\ 11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2,\ -11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2\right\}$$

In[23]:=
```
EZ[gd]
```

Out[23]=
$$\frac{xX\left(9-22X+27X^2-12X^3\right)-2DR\left(3-9X+11X^2-9X^3+3X^4\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[24]:=
```
EZ1[gd]
```

Out[24]=
$$\left\{DR+\frac{x\left(-1+5X-8X^2+8X^3-5X^4+X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ DR+\frac{x\left(-1+5X-8X^2+8X^3-6X^4+2X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\right.$$
$$-DR-\frac{x\,(-1+X)^2\left(1-X-X^2+3X^3\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ -DR-\frac{x\left(8+\frac{1}{X^3}-\frac{1}{X^2}-\frac{4}{X}-8X+3X^2\right)}{11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2},$$
$$-DR-\frac{x\,(-1+X)^2\left(-2+4X-3X^2+3X^3\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},\ DR-\frac{x\left(1-3X+3X^2-2X^3+X^4\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)},$$
$$-DR-\frac{x\left(-1+3X-3X^2-X^3+X^4\right)}{3-9X+11X^2-9X^3+3X^4},\ DR+\frac{x\left(6+\frac{1}{X^3}-\frac{4}{X}-5X+X^2\right)}{11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2},$$
$$\left.-DR-\frac{x\left(2-6X+8X^2-8X^3+3X^4\right)}{3-9X+11X^2-9X^3+3X^4},\ -DR-\frac{x\left(-2+8X-14X^2+16X^3-11X^4+3X^5\right)}{X\left(3-9X+11X^2-9X^3+3X^4\right)}\right\}$$

In[25]:=
```
Simplify[x X D[Log[Alexander[K][X]], X]]
```

Out[25]=
$$\frac{3x\left(-2+3X-3X^3+2X^4\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[26]:=
```
SL[K]
```

Out[26]=
$$-2\,DL$$

In[27]:=
```
EZ[gd] /. DR → 0
```

Out[27]=
$$\frac{x\,X\left(9-22X+27X^2-12X^3\right)}{3-9X+11X^2-9X^3+3X^4}$$

In[28]:=
```
n = Length[gd]; R[gd] // MatrixForm
```

Out[28]//MatrixForm=
$$\begin{pmatrix}
1-X & 0 & -1 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 & 0 \\
0 & 1-\frac{1}{X} & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\
-1 & \frac{1}{X} & 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 \\
0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 & 0 \\
0 & 0 & 0 & 1-X & 0 & 0 & 0 & 0 & -1 & X & 0 \\
0 & -1 & \frac{1}{X} & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & X & 0 & 0 & 0 & 1-X & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 & 0 & 1-\frac{1}{X} \\
0 & 0 & 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & -1 & \frac{1}{X}
\end{pmatrix}$$

In[29]:=
```
r = R[gd]; r[[All, 1]] += r[[All, n + 1]]; r = Drop[r, None, -1];
MatrixForm[r]
```

Out[30]//MatrixForm=

$$\begin{pmatrix} 1-X & 0 & -1 & X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 \\ 0 & 1-\frac{1}{X} & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 & 0 & 0 \\ -1 & \frac{1}{X} & 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 \\ 0 & 0 & 0 & 1-X & 0 & 0 & 0 & 0 & -1 & X \\ 0 & -1 & \frac{1}{X} & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & X & 0 & 0 & 0 & 1-X & 0 \\ 1-\frac{1}{X} & 0 & 0 & 0 & 0 & -1 & \frac{1}{X} & 0 & 0 & 0 \\ \frac{1}{X} & 0 & 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[31]:=
```
Det[r]
```

Out[31]= 0

In[32]:=
```
Det[Drop[ r, -1, -1]]
```

Out[32]= $-11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2$

In[33]:=
```
Alexander[K][X]
```

Out[33]= $-11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2$

In[34]:=
```
Union @@ Minors[r]
```

Out[34]= $\left\{9-\frac{3}{X^3}+\frac{9}{X^2}-\frac{11}{X}-3X,\ -9+\frac{3}{X^3}-\frac{9}{X^2}+\frac{11}{X}+3X,\ -11-\frac{3}{X^2}+\frac{9}{X}+9X-3X^2,\ 11+\frac{3}{X^2}-\frac{9}{X}-9X+3X^2\right\}$

In[35]:=
```
Union[Expand[X^(-Exponent[#, X, Min]) #] & /@ Union @@ Minors[r]]
```

Out[35]= $\left\{-3+9X-11X^2+9X^3-3X^4,\ 3-9X+11X^2-9X^3+3X^4\right\}$

In[36]:=
```
PolynomialGCD @@ Union[Expand[X^(-Exponent[#, X, Min]) #] & /@ Union @@ Minors[r]]
```

Out[36]= $3-9X+11X^2-9X^3+3X^4$