

The Weight System of the MVA

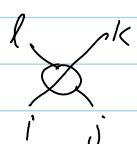
August-01-10
9:51 AM

Some Convention:

$$X_p : \begin{array}{c} k \\ \nearrow \\ l \end{array} \quad X_m : \begin{array}{c} l \\ \nearrow \\ i \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{from the Sandbjerg handout.}$$

$$\begin{array}{c} k \\ \nearrow \\ i \\ \parallel \\ j \end{array} \rightarrow \begin{array}{c} l \\ \nearrow \\ j \end{array} : \text{ar}[i, j, k, l] = \deg_1 \text{ of } X_p[i, k, l, i] - I$$

so it is



$$\text{pA[SVXp[i, j, k, l]] := AHD[} \\ (\text{t}[i] == \text{t}[k]) (\text{t}[j] == \text{t}[l]), \{i, j\}, W[k, l], \\ -W[i, k] + t[j] W[i, k] + W[i, l] - t[i] W[i, l] - W[k, l] + t[i] W[k, l] \\]$$

$$\text{So } \begin{array}{c} k \\ \nearrow \\ i \\ \parallel \\ j \end{array} \rightarrow \begin{array}{c} l \\ \nearrow \\ j \end{array} = \begin{array}{c} l \\ \nearrow \\ i \\ \parallel \\ j \end{array} - \begin{array}{c} l \\ \nearrow \\ i \\ \parallel \\ k \end{array} + \begin{array}{c} l \\ \nearrow \\ i \\ \parallel \\ l \end{array}$$

$$\begin{array}{lll} aa = 0 & ba = t_1 a & ca = 0 \\ ab = 0 & bb = t_1 b & cb = 0 \\ ac = 0 & bc = 0 & cc = t_1 c \end{array} \quad (a-b+c)^n = (-b)^{n-1}(a-b) + c^n$$

upto signs

$$\begin{array}{c} j \\ \nearrow \\ i \end{array} \rightarrow 0 := \begin{array}{c} j \\ \nearrow \\ l \\ \parallel \\ k \\ \nearrow \\ i \end{array} - \begin{array}{c} j \\ \nearrow \\ l \\ \parallel \\ i \end{array} = x_j \begin{array}{c} l \\ \nearrow \\ i \end{array} !$$

PA doesn't behave well under strand reversals!

Possibly, there is no "canonical" extension of MVA to w-knots, so there's no point looking for its weight system.

Added Feb 20, 2012: was

This red statement about w-knots or about w-tangles?

The Archibald relation:

Perhaps there is a canonical wMVA for knots yet not for tangles?

$$\begin{array}{c} l \\ \nearrow \\ m \\ \nearrow \\ n \\ \parallel \\ p \end{array} - \begin{array}{c} l \\ \nearrow \\ m \\ \nearrow \\ n \\ \parallel \\ p \end{array} = -x_j^i \begin{array}{c} l \\ \nearrow \\ m \\ \nearrow \\ n \\ \parallel \\ p \end{array} + x_i^j \begin{array}{c} l \\ \nearrow \\ m \\ \nearrow \\ n \\ \parallel \\ p \end{array}$$

$$\left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) - \left(\begin{array}{c|c|c} j_1 & j_2 & j_3 \\ \hline k_1 & k_2 & k_3 \end{array} \right) = -x_j^i \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) + x_i^j \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline k_1 & k_2 & k_3 \end{array} \right)$$

Is there a "dimensional relation" like

$$\left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) = 0 \quad ? \quad \begin{matrix} \text{(so far)} \\ \text{no} \end{matrix}$$

The "go with the flow" W.S.:

$$\left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) \stackrel{\text{OC}}{=} \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right)$$

$$4T? \quad \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) + \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) \stackrel{?}{=} \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right) + \left(\begin{array}{c|c|c} i_1 & i_2 & i_3 \\ \hline j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{array} \right)$$

Conjecture WMA is supported on "funnel" diagrams, and its value is ± 1 according to the parity of the number of "strand hops".

with x_i^j 's determined by tails.

That and on one of their two starting strands.

False! According to WSofMVA.nb. $? \text{ No } 4T \text{ yet}$

Important The WS of the MVA reduces degree by 1: $W(\text{degree } k \text{ diagram}) = (\text{polynomial of deg } k-1)$. In particular, it seems that the constant term

of the MVA is the total linking number.

Q Could it be that W is defined via

