Let $Z$ be a group-like valued UFTI of something. Then:

1. Is $\log Z : K \rightarrow \mathcal{PA}$ filtered?
2. What is the weight system $\text{Weg}_Z$ of $\log Z$? It would be a map

$$\text{Weg}_Z : K \rightarrow \mathcal{PA}.$$ 

Why bother? It would be nice to understand $\ker \text{Weg}_Z$, as it is possibly related to the non-group-like Alexander relations.

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Added July 30, 2010: And if we're at it, what's the weight system of $Z^{-1}\mathbb{E}Z$? i.e., of $K \rightarrow \mathbb{Z}(K)^{-1}\mathbb{E}Z(K)$?

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What's the weight system of $Z^{Z_0}$?

$$K \rightarrow K \otimes K \xrightarrow{\text{red}} A \otimes A \xrightarrow{\eta} A$$

$$A \xrightarrow{\circ} A \otimes A \xrightarrow{1 \otimes 1} A \otimes A \xrightarrow{\eta} A.$$

So $\text{Weg} Z = \text{M} \circ \text{W}$. 

$$\log Z = \sum_{n=1}^{\infty} \frac{(1-z)^n}{n}.$$ 

\[\text{diff:} \quad \frac{1}{2} \sum_{n=1}^{\infty} \frac{z(1-z)^n}{n2^n} = \frac{1}{1-(1-z)^2} = \frac{1}{2z} \]

\[\text{\text{Weg}(log[z]) = \{n,1,5\}} \]

\[\text{opp:} \quad \frac{1}{2}z(1-z)^2 \frac{1}{3}z(1-z)^3 \frac{1}{4}z(1-z)^4 \frac{1}{5}z(1-z)^5 \frac{1}{6}z(1-z)^6 \frac{1}{7}z(1-z)^7 \frac{1}{8}(z-1)^8 \]

So $\text{Weg}_Z(D) = \text{sum over all non-trivial splits of } D$, the product of the split edge prices $1 \cdots 1 \cdots 8$. 

2010-07  Page 1
divided by their number | & multiplied by $(-1)^{|H|}$. 

Example: $x \mapsto x$  

| $xy \mapsto xy - \frac{1}{2} xy = 0$ | $1-1 = 0$ |
| $xy_2 \mapsto xy_2 - \frac{1}{2} 2 \times xy_2 + \frac{1}{3} 6 xy_2 = 0$ | $1-3+2=0$ |
| $xy_{2w} \mapsto 1 - \frac{1}{2} 14 + \frac{1}{3} 6 \cdot 3 \cdot 2 - \frac{1}{4} 2 y = 1-7+12-6 = 0$ |
| $a_{y_{-1}} \mapsto 1 - \frac{1}{2} 30 + \frac{1}{3} (6 \cdot 2 + 5 \cdot 3) - \frac{1}{4} (10 \cdot 3) + \frac{1}{5} \cdot 120$ |
| $= \frac{3^0 - 2^3 + 3 \cdot 150}{240}$ |
| $= 243-24+45=150$ |
| $= 1 - 5+50-60+2y = 0$ |

Can this be categorized? 

Example: (The non-Abelian case):  

$X \mapsto x$  

| $xy \mapsto xy - \frac{1}{2} (xy + xy) = \frac{1}{2} [xy]$ |
| $xy+yx \mapsto 0$ etc. |

Can I recover $W_{\text{Kw}}=W_{\text{Kd}}$ from this? 

No. All that we've learned is that there is a universal projection $A \to P$, simply by mapping all higher symmetric products of primitives to 0. But that was obvious to start with, and it probably has no good algebraic properties. (E.g., it is not the quotient by an internal relation).

So what's $W_{\text{Kw}}=W_{\text{Kd}}$? Is there some other way to say "it's the internal relation that collapses everything into the primitives"?