Let Z be a group-like-valued UFTI of something. Then:

1. Is $\log Z : K \to \mathcal{PA}$ filtered?
2. What is the weight system $W_{\log Z}$ of $\log Z$? It would be a map
$$W_{\log Z} : \mathcal{A} \to \mathcal{PA}.$$

Why bother? It would be nice to understand $\ker W_{\log Z}$, as it is possibly related to the non-group-like Alexander relations.

Added July 30, 2010: And if we're at it, what's the weight system of $Z^{-1}EZ$; i.e., of $K \mapsto Z(K)^{-1} E Z(K)$?

What's the weight system of Z^2?

$$K \xrightarrow{\square} K \otimes K \xrightarrow{Z \otimes Z} \mathcal{A} \otimes \mathcal{A} \xrightarrow{m} \mathcal{A}$$

$$\mathcal{A} \xrightarrow{\square} \mathcal{A} \otimes \mathcal{A} \xrightarrow{I \times I} \mathcal{A} \otimes \mathcal{A} \xrightarrow{m} \mathcal{A}$$

So $W_{Z^2} = m \circ \square$.

$$\log z = \sum_{n=1}^{\infty} \frac{(1-z)^n}{n}$$

In[1]:= Series[Log[z], {z, 1, 5}]

Out[1]= $(z-1) - \frac{1}{2}(z-1)^2 + \frac{1}{3}(z-1)^3 - \frac{1}{4}(z-1)^4 + \frac{1}{5}(z-1)^5 + O[z-1]^6$

diff:
$$\frac{1}{z} \overset{?}{=} \sum_{n \geq 1} (1-z)^{n-1} = \sum_{n \geq 0} (1-z)^n = \frac{1}{1-(1-z)} = \frac{1}{z}$$

So $W_{\log Z}(D) =$ Sum over all non-trivial splits of D, the product of the splitted pieces [illegible] divided by their number n & multiplied by $(-1)^{n+1}$.

Example: $x \mapsto x$ $\qquad 1 \qquad$ (The Abelian case)

$$xy \mapsto xy - \tfrac{1}{2}2xy = 0 \qquad 1-1=0$$

$$xyz \mapsto xyz - \tfrac{1}{2}3\cdot 2\,xyz + \tfrac{1}{3}6xyz = 0 \qquad 1-3+2=0$$

$$xyzw \mapsto 1 - \tfrac{1}{2}14 + \tfrac{1}{3}6\cdot 3\cdot 2 - \tfrac{1}{4}24 = 1-7+12-6=0$$

$$a_1\ldots a_5 \mapsto 1 - \tfrac{1}{2}30 + \tfrac{1}{3}\big(\overset{311}{10\cdot 3\cdot 2} + \overset{221}{5\cdot 6\cdot 3}\big) - \tfrac{1}{4}\big(\overset{2111}{\underset{240}{10\cdot 4\cdot 6}}\big) + \tfrac{1}{5}\cdot 120$$

$$= 3^5 - 3\cdot 2^5 + 3 = 150$$

$$= 243 - 96 + 3 = 150$$

Can this be categorified?

$$= 1 - 15 + 50 - 60 + 24 = 0$$

Example (The non-Abelian case):

$$x \mapsto x$$

$$xy \mapsto xy - \tfrac{1}{2}(xy+yx) = \tfrac{1}{2}[x,y]$$

$$xy+yx \mapsto 0 \qquad \text{etc.}$$

Can I recover $W_k W_l = W_{k+l}$ from this?

No. All that we've learned is that there is a universal projection $A \to P$, simply by mapping all higher symmetric products of primitives to 0. But that was obvious to start with, and it probably has no good algebraic properties. (E.g., it is not the quotient by an internal relation).

So what's $W_k W_l = W_{k+l}$? Is there some other way to say "it's the internal relation that collapses everything into the primitives"?