

What do we have to change in order to do QFT on non-commutative space time?

Question - What is locality?

Toy model: Moyal plain: $[X^\nu, X^\mu] = \Theta^{\nu\mu}$ central.

May assume $\Theta^{\mu\nu}$ is a multiple of I .

often

$$\Theta^{\mu\nu} = \begin{pmatrix} 0 & -a & 0 \\ a & 0 & 0 \\ 0 & 0 & b \\ & & -b & 0 \end{pmatrix}$$

$SO(1,3)$

↓

$SO(1,1) \times SO(2)$

(Question - to what extent is the smaller group good enough?)

The Wightman Axioms:

\mathcal{H} a Hilbert space, $\Omega \in \mathcal{H}$ "vacuum",

$\mathbb{R}^4 \times SO_+(1,1) \times SO(2)$ acts on \mathcal{H} with $\mathcal{H}^{\mathbb{R}^4} = \mathbb{C}\Omega$,

spectral measure of translations is contained in the forward light cone.

Fields are distributions with values operators on \mathcal{H} .