What do we have to change in order to do QFT on non-commutative space-time?

Question - What is locality?

Toy model: Moyal plain: \([x^\nu, x^\mu] = \Theta^{\nu\mu}\) central.

May assume \(\Theta^{\mu\nu}\) is a multiple of \(i\).

Often

\[
\Theta^{\mu\nu} = \begin{pmatrix}
0 & a & 0 \\\na & 0 & b \\
0 & b & 0
\end{pmatrix}
\]

\(SO(1,3) \downarrow SO(1,1) \times SO(2)\)

(Question - to what extent is the model good enough?)

The Wightman Axioms:

\(\mathcal{H}\) a Hilbert space, \(\mathcal{H}^0\) "vacuum",

\(\mathbb{R}^4 \times SO_+(1,1) \times SO(2)\) acts on \(\mathcal{H}\) with \(d^\mu\nu = \varepsilon_{\mu\nu}\),

spectral measure of translations is contained in the forward light cone.

Fields are distributions with values operators on \(\mathcal{H}\).