Abstract. I will define w-knots, a class of knots wider than ordinary knots but narrower than virtual knots, and show that it is quite easy to construct a universal finite invariant \( Z \) of w-knots. In order to study \( Z \) we will introduce the "Euler Operator" and the "Infinitesimal Alexander Module", at the end finding a simple determinant formula for \( Z \). With no doubt that formula computes the Alexander polynomial \( A \), except I don't have a proof yet.

The Alexander Theorem. For a w-knot \( \mathbf{K} \), \( \mathbf{K} \) is determined by \( A \).

The Bracket-Rise Theorem. \( A^* \) is isomorphic to \( 1+1+1 \) and \( \text{IIX} \) relations.

Corollary. (1) Related to Lie algebra. (2) Only wheel, and isolated arrow point.

The Alexander Theorem. \( A \) is determined by \( \mathbf{K} \).

Conjecture. For a w-knot \( \mathbf{K} \), \( A \) is the Alexander polynomial.

Theorem. With \( u : \alpha^k \rightarrow \alpha^k \) (the \( k \)-twist).

Proof Sketch. Let \( E \) be the Euler operator, "multiply anything by degree", \( f \rightarrow f^k \) in \( \mathbf{Q}[\alpha] \), so \( E^n \rightarrow \alpha^n \).

We need to show that \( Z^n \rightarrow Z^n, (\alpha - \beta) \rightarrow (\alpha - \beta)^n \).

Note that \( Z^n \rightarrow Z^n, (\alpha - \beta) \rightarrow (\alpha - \beta)^n \).

With matrices \( X \) and \( Y \) defined as

\[
\begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix}
\]

we have \( E^2 \rightarrow \alpha^n \), \( A = -BY + T_{\alpha}^{\alpha^2} \), and \( Y = BY + T_{\alpha}^{\alpha^2} \). The theorem follows.

For more:
- Habiro-Shima (to be submitted)
- JSJ: Jones-Samelson-Habiro-
- Bar-Natan (to be submitted)
- An "Alexander invariant": local, compact, well-behaved under cobbling. Could be used to generalize the multi-variable Alexander polynomial and the theory of Milnor linking numbers.
- Top of the (Kanen瓦-Veigele) conjecture and Drinfeld's conjecture.
- See also http://www.math.mcmaster.ca/~drols/papers/WEQ/

"I like more knots, and less in topology in the work of mirrors."

S. Keilin (outdated)

w-Knots from Z to A

Dror Bar-Natan, Luminy, April 2010

http://www.math.toronto.edu/~drorbn/Luminy-2010/
w-Knots from Z to A
Doron Bar-Natan, Luminy, April 2010
http://www.math.toronto.edu/~doronb/Talks/Luminy-0404/Abstract
I will define w-knots, a class of knots wider than ordinary knots but narrower than virtual knots, and show that it is quite easy to construct a universal finite invariant Z of w-knots. In order to study Z we will introduce the “Euler Operator” and the “Infinitesimal Alexander Module”, and at the end find a simple determinant formula for Z. With no doubt that formula computes the Alexander polynomial \( A \), except I don’t have a proof yet.

A Ribbon 2-Knot is a surface \( S \) embedded in \( \mathbb{R}^4 \) that bounds an immersed handlebody \( B \), with only “ribbon singularities”: a ribbon singularity is a disk \( D \) of transverse double points, whose preimages in \( B \) are a disk \( D_1 \) in the interior of \( B \) and a disk \( D_2 \) with \( D_2 \cap \partial B = \partial D_2 \), modulo isotopies of \( S \) alone.

w-Knots
\[
\begin{align*}
\text{wK} & = \text{CA} / \text{VR23}, \text{OC} \\
& = \text{PA} / \text{VR23}, \text{VR123}, \text{D}, \text{OC}.
\end{align*}
\]

The F.I.T. Story

So what?
* The ultimate Alexander invariant: local, composes well, behaves under cabling.
* Tip of the Alexander-Michelson icing.
* Tip of the w-knots icing.

[Diagram of w-knots and related concepts]