Question. Is it possible to repeat the EK argument in the w case, with $U(g^+)$ replacing $M^+$?

In a general bi-algebra, there's only one way to map $U(g) \rightarrow U(g_+)$ or $U(g) \rightarrow U(g_-)$, and this is via $M_+ \& M_-$ [i.e., using PBW followed by a projection]. The "other" map $U(g) \rightarrow U(g_+)$ that exists in the w-case is a coincidence.

Note: In the w case, $g_+ \otimes g_- \rightarrow g_+$ is a Lie alg. map, but $g_+ \otimes g_- \rightarrow g_-$ isn't.