Whence BCH?

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1:19 AM

Let G be a group and Q_G its quandle.
What would be a homomorphic expansion for

$$\overset{\wedge}{\circlearrowleft} Q_G \hookrightarrow G \overset{\times}{\circlearrowleft} \ ?$$

How about $\overset{\wedge}{\circlearrowleft} FQ(x,y) \longrightarrow FG(x,y) \overset{\times}{\circlearrowleft}$?

In particular, what if $G=F(x,y)$ is the free group on two generators?

Even before, what's a homomorphic expansion for $Q=FQ(X,Y)$, the free quandle on two generators?

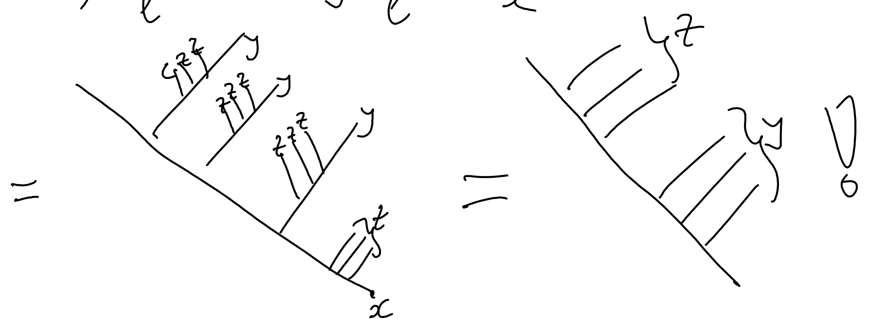
$X \longrightarrow x=[X-1]$ $\qquad$ $Y \longrightarrow y=[Y-1]$

$X \wedge Y \longrightarrow \log(e^{-y}e^{x}e^{y}) = e^{-y}xe^{y} = e^{-\operatorname{ad} y}x$

$(X\wedge Y)\wedge Z \longrightarrow e^{-\operatorname{ad} z}(e^{-\operatorname{ad} y}x)$

$(X\wedge Z)\wedge(Y\wedge Z) \longrightarrow e^{-\operatorname{ad}(e^{-\operatorname{ad} z}y)}e^{-\operatorname{ad} z}x$

$X\wedge Y-1=$
$(X-1)\wedge 1+X\wedge(Y-1)$
So x, y generate proj Q



$\Longrightarrow$ So, it seems, a homomorphic expansion for $Q(X,Y)$ exists with no need for the BCH formula.

Project: Keep pushing to the point that BCH becomes necessary.

What's a homomorphic expansion for $Q = FG(x,y) = G$ the group-quandle of the free group on 2 generators?

Note As a quandle, Q is generated by minimal representatives of conjugacy classes of G. Is it free?