\[(x \land y) \land z = (x \land z) \land (y \land z)\]  

\[\{ = (x \land (y \land z)) \land (x \land (y \land z)) = \ldots \}\]

\[x = y, z = 1 \rightarrow x \land x = x \land x\]
\[x = z, y = 1 \rightarrow x \land x = (x \land x) \land 1 = x \land x\]
\[x = 1, y = z \rightarrow 1 = 1 \land (y \land z) = 1\]

... It doesn't seem like \[x \land x = x\] follows from \[\land x = 1\] & \[x \land 1 = x\].

Is it needed? \[\quad always, x \land 1 = z \land x \land 1 = 0\]

\[0 = [(1 + x) \land (1 + x)] \land (1 + x) - [(1 + x) \land (1 + x)] \land [(1 + y) \land (1 + z)]\]

\[= 1 - 1 \quad \text{deg 0}\]
\[+ x - \bar{x} \quad \text{deg 1}\]
\[+ x \bar{y} + x \bar{z} - \bar{x} y - \bar{x} \bar{z} \quad \text{deg 2}\]
\[+ (x + y) \bar{z} - \bar{x} (y + z) - (x \bar{z} + y \bar{z}) \quad \text{deg 3}\]
\[+ (x \land \bar{z}) \land (y \land z) \quad \text{deg 4}\]

No more for \[x \land x = x\] up to here?

However it is needed in order to show that the bracket is anti-symmetric, \[x \land \bar{x} = 0\]:

\[0 = x \land \bar{x} - x = (1 + x) \land (1 + x) - (1 + x)\]

\[= 1 - 1 \quad \text{deg 0}\]
\[+ x - \bar{x} \quad \text{deg 1}\]
\[+ x \bar{y} + x \bar{z} - \bar{x} y - \bar{x} \bar{z} \quad \text{deg 2}\]

If we do not impose \[x \land x = x\], do we get a “Lobnitz algebra”?  

\underline{Questions} If an algebra is Lie at the level of generators is it Lie in general?
generators, is it Lie in general?

\[ [[a,b], [a, b]] = V \\ V \\

\[ x^{\Lambda} y - x^{\Lambda} y = x^{\Lambda} (y - y) + (x - x) y = x^{\Lambda} 1 + 1 y = x \\
L^c = (x-x) y + \overline{x} (y-\overline{y}) = 1 y + \overline{x} 1 = 1 + \overline{x} = x \]

**Experiment left.**

\[ ((x^{\Lambda} y)^{\overline{z}})^{\Lambda w} = (x^{\Lambda} y)^{1 w} \wedge (z^{\Lambda} w) \]

so

\[ ((x^{\Lambda} y + x) \wedge (z^{\Lambda} z + 1)) \wedge (w^{\Lambda} + 1) = ((x^{\Lambda} y + x) \wedge (w^{\Lambda} + 1)) \wedge (z^{\Lambda} z + 1) \]

deg 0: \( (x^{\Lambda} y)^{1} = (x^{\Lambda} y)(1 1) \) ✔

deg 1: \( x^{\Lambda} z + x^{\Lambda} \overline{y} = x^{\Lambda} \overline{w} + x^{\Lambda} z \) ✔

deg 2: \( x^{\Lambda} \overline{y} + (x^{\Lambda} z) \wedge \overline{w} = x^{\Lambda} y + (x^{\Lambda} w) \wedge \overline{z} + x^{\Lambda} (z^{\Lambda} \overline{w}) \)

**Aside** \( (x^{\Lambda} y)^{\overline{z}})^{\Lambda w} = (x^{\Lambda} y)^{1 (y + 1)} \wedge w = (x^{\Lambda} y)^{1 (y + 1)} \wedge w = \ldots \)

\[ (w^{\Lambda} x)^{1 (y^{\Lambda} z)} = (w^{\Lambda} (y^{\Lambda} z)) \wedge (x^{\Lambda} (y^{\Lambda} z)) \] so

\[ [(1 + \overline{w})^1 (1 + x)] \wedge (\overline{y}^{\Lambda} z + \overline{y} + 1) = [(1 + \overline{w})^1 (y^{\Lambda} z + \overline{y} + 1)] \wedge (1 + \overline{w})^1 (y^{\Lambda} z + \overline{y} + 1) \]

deg 0: \( 1 = 1 \) ✔

deg 1: \( \overline{w} = \overline{w} \) ✔

deg 2: \( \overline{w} \wedge x + \overline{w} \overline{y} = \overline{w} \overline{y} + \overline{w} x \) ✔

deg 3: \( \overline{w} \wedge (y^{\Lambda} z) + \overline{w} x \wedge y = \overline{w} (y^{\Lambda} z) + (\overline{w} x) y + (\overline{w} x) y \)

deg 4: \( \overline{w} \wedge (y^{\Lambda} z) = (\overline{w} \wedge (y^{\Lambda} z)) \wedge x + \overline{w} \wedge (x^{\Lambda} (y^{\Lambda} z)) + (\overline{w} x) \wedge (y^{\Lambda} z) \)

The error from degree 3 fixes degree 4, so Jacobi holds.
Yes 1. Could this have been done in express that?
2. Does this generalize?

Experiment left, redo:

$$((wx)^{y+1})^2 = ((wx)^{y+1})^2 + (y+1)$$

So

$$[[wx + \bar{w} + 1]^y (z+1)]^y (z+1) = (\bar{w}x + \bar{w} + 1)^y (z+1)]^y (\bar{y} + 1) (z+1)$$

deg 1:

$$\bar{w} = \bar{w}$$

deg 2:

$$\bar{w}x + \bar{w}y + \bar{w}z \bar{y} = \bar{w}x + \bar{w}y + \bar{w}z \bar{y}$$

deg 3:

$$[wx]^y (\bar{wx} + \bar{w}y + \bar{w}z \bar{y}] = (\bar{wx})^y (\bar{w}x + \bar{w}y + \bar{w}z) + (\bar{w}z)$$

deg 4:

$$[wx]^y (\bar{w}x + \bar{w}y + \bar{w}z) = (\bar{wx})^y (\bar{w}x + \bar{w}y + \bar{w}z)$$

Works in the same way as experiment right.... As it should, or else proj(free quandle) would not be (free Lie).

Let $[xy] := x\bar{y} - x$ or $x\bar{y} = x + [x, y]$.
We have $[x, \bar{y}] = x\bar{y} - x = x\bar{y} - 1$

in the bilinear interp, $[x, \bar{y}] = x\bar{y}$
and also $[x, 1] = 0$, $[1, \bar{x}] = 0$

$$(xy)^{y+1} = (x+1)^y (y+1) \Rightarrow$$

$$[xy,xz] + [xy, yz] = [x,y] + [y,x] + [z,x] + [z,y]$$

$$\Rightarrow [xy, z] = [x,z] + [y,z] + [x,y]z$$

$$\Rightarrow$$

same with $x, \bar{y}, z$. [I may have to introduce a co-product to deal with $z$.]
\[ \Delta (\overline{x} \cdot \overline{y}) = \Delta (\overline{x} \cdot \overline{y} - x \cdot y) = (x \cdot y) \otimes (x \cdot y) - x \otimes x \]
\[ = (x \cdot y) \otimes (x \cdot y) + x \otimes (x \cdot y) = (x \cdot y + 1) \otimes (x \cdot y) + R \]
where
\[ R = (x \cdot y) \otimes (x \cdot y - 1) + (x - 1) \otimes (x \cdot y) = (x \cdot y) \otimes (x \cdot y + x) + x \otimes (x \cdot y) \]

It seems that we need the following lemma:

**Lemma:** If \( z \in I^m \), then \( \Delta z - (z \otimes 1 + 1 \otimes z) \in \bigoplus_{k=0}^{m} I^k \otimes I^l \) (for \( m \geq l \)).

\[ \Delta X = X \otimes X \quad \Delta \overline{x} = X \otimes X - 1 \otimes 1 = X \otimes X + 1 \otimes 1 \]
\[ = \overline{x} \otimes 1 + 1 \otimes \overline{x} + \overline{x} \otimes \overline{x} \]
\[ \Delta (\overline{x} \cdot \overline{y}) = (X \otimes X + 1 \otimes 1) \cdot (X \otimes X + 1 \otimes 1) \]
\[ = (X \otimes X + 1 \otimes 1) \otimes (X \otimes X + 1 \otimes 1) \]
\[ = (X \otimes X) \otimes (X \otimes X) + (X \otimes X) \otimes (X \otimes X) + \overline{x} \otimes (X \otimes X) \]
\[ + (\overline{x} \otimes X) \otimes (X \otimes X) \]

**Claim:** With \( u : ZQ \otimes ZQ \to ZQ \) the linear extension of \( x \otimes y \mapsto x \cdot y \), and with \( \Delta : ZQ \to ZQ \otimes ZQ \) the linear extension of \( x \mapsto x \otimes x \), we have

\[ \Delta \circ u = (u \otimes u \circ (X \otimes X \circ \Delta)): ZQ \otimes ZQ \to ZQ \otimes ZQ \]

**Proof:** Both sides of the equation are linear maps, and on \( (x \cdot y) \), both give \( (x \cdot y) \otimes (x \cdot y) \).

**Proof of lemma:** By induction on the structure of \( Z = Z \otimes Z \).

Take \( z_i \in I^m \) for \( m \geq 1 \), \( i = 1, 2 \). Then

\[ \Delta (z) = \Delta (u \cdot (z_1 \otimes z_2)) = u \cdot (\Delta (z_1) \otimes \Delta (z_2)) = \]
\[ = u \cdot (z_1 \otimes z_1 + z_2 \otimes z_2) \otimes (z_1 \otimes z_1 + z_2 \otimes z_2) \]
\[ = (z_1 \otimes z_2) \otimes (z_1 \otimes z_2) \]
\[
\frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) + \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) + \frac{1}{\sqrt{2}} \left( \begin{array}{c} -1 \\ 1 \end{array} \right)
\]

It remains to show that the bracket is \textbf{AS} in general \( \triangledown \). \( x = x \times \Rightarrow \)

\[
z = u p ( z \circ 1 + 1 \circ z + z \circ z )
\]

\[= z + \sum z', \circ z''
\]

Only gives as the uncontrolled \( \sum z'' = 0 \).

\[
\text{Question If a f,g. algebra satisfies } \textbf{Jacobi} \text{ and } x \times x = 0 \text{ for every generator } x, \text{ does it follow that it is a Lie algebra } ?
\]

\[
[x, y], [x, j] = \left[ \left[ x, y \right], x \right] + \left[ x, \left[ x, j \right], j \right] = ?
\]

Even earlier, can we show that \( [x, [x, y]] = - [[x, j], x] \) ? Yes.

\[
[x, [y, z]] = \sum_{x} [x, [y, z]] = [x, [y, z]] + [x, [x, z]]
\]

\[= - [x, [y, x]] - [y, [x, z]]
\]

\[= - [x, [y, x]] - [y, [x, z]] - [y, [x, z]] - [y, [x, z]]
\]

\[= - [x, [y, x]] - [y, [x, z]] - [y, [x, z]] - [x, [y, z]]
\]

\[= - [y, [x, z]] - [x, [y, z]] - [x, [y, z]]
\]

This implies \( [x, [y, z]] = 0 \) by setting \( x \times x = 0 \).

\[
\Delta x = x \circ 1 + 1 \circ x + x \circ x.
\]

\( \text{Does this persist? No:} \)

\( \text{Assume true for } z, z. \text{ Test for } z, z: \)

\[
\Delta (z, z) = (\Delta z)^{(1)}(0 z_2) =
\]

2010-03 Page 5
\[
(z_0 + \alpha_1 z_1 + \alpha_2 z_2)^\land (z_0 + \alpha_1 z_1 + \alpha_2 z_2) = \\
= (z_1 z_2) o 1 + 1 o (z_1 z_2) + (z_1 z_2) \otimes z_1 + z_0 (z_1 z_2) \\
+ (z_1 z_2) \otimes (z_1 z_2)
\]

Yet from the above, with \(z_1 z_2 = \text{wr}(\Delta(z_1 z_2)) = z_1 z_2 + (z_1 z_2) 1 z_1 + z_0 (z_1 z_2)\), we have AS for \(z_1 z_2\).

[This holds only for \(z_1 z_2 \in I\) anyway.]

Can we likewise prove that always things obey AS with their conjugates? Would this be enough? Can we prove that \(z_1\) is AS with \(z_1 \wedge z_3\)?

In general,
\[
\text{wr}(\Delta(z_1 + z_2)) = 
\]