Proj Q, For Honest Quandles

March-04-10
11:46 AM

Following Joyce, an honest quandle Q is a set Q with two binary ops $\triangleright$ & $\triangleright^{-1}$, s.t.

1. $x \triangleright x = x$
2. $(x \triangleright y) \triangleright^{-1} y = x$
3. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$.

Rename $\triangleright = \wedge$, $\triangleright^{-1} = \vee$

Question – what's proj Q?

Let $\bar{x} := x - y$. Then

$(x-y) \wedge (x-y) = x + y - x \wedge y - y \wedge x$

$0 = (y+\bar{x}) \wedge (y+\bar{x}) - (y+\bar{x})$

$= y \wedge y - y$ deg 0

$+ y \wedge \bar{x} + \bar{x} \wedge y - \bar{x}$ deg 1

$+ \bar{x} \wedge \bar{x}$ deg 2

$y \wedge x - y + x \wedge y - y - x + y$

$= y \wedge x + x \wedge y - y - x$