starting idea in statistical mechanics:

Instead of looking at individual phase points,
study probability measures on phase space; replace
physical quantities by their means.

Boltzmann/Gibbs: The appropriate measure
has density prop. to $e^{-\beta H(x)}$.

This measure will be invariant under the physics flow
$\beta = \frac{1}{kT}$, when $T$ is the absolute temperature
and $k$ = Boltzmann's constant =

Average kinetic energy per particle = $\frac{1}{2} kT$

This is "the canonical ensemble".

\[
\text{Notation: } x = (\vec{p}_1, \ldots, \vec{p}_N) \\
\text{Probability measure: } C e^{-\beta H(x)} \frac{d\vec{q}_1 \cdots d\vec{p}_N}{N!}
\]

Conventionally, $C = \frac{1}{Z}$ where $Z = "the \ canonical \ partition \ function"

\[
Z = \int e^{-\beta H(x)} \, dx
\]

The equilibrium value of the energy (as a function
of $\beta$ or $T$)

\[
< H > _\beta = \frac{1}{Z} \int H e^{-\beta H(x)} \, dx \\
= \frac{1}{Z} \left( \frac{\partial}{\partial \beta} \right) \int e^{-\beta H(x)} \, dx = -\frac{1}{Z} \frac{\partial}{\partial \beta} (\log Z)
\]
\[ = \frac{1}{2} \left( \frac{d}{dp} \right) \left( e^{-\beta H(p)} \right) = \frac{d}{dp} \left( \log Z \right) \]

Integrals over \( p \) are more or less trivial, as they are Gaussians. So
\[
Z = \left( \frac{2\pi m}{\beta} \right)^{N/2} \int e^{-\beta U(\mathbf{p})} \frac{d\mathbf{p}}{N!} \quad (m \text{ is the mass of a particle})
\]

Let \( \mathbf{p}_{ij} \) be the \( ij \)th component of \( \mathbf{p} \). Then
\( \mathbf{p}_{ij} \) are independent Gaussian random variables
with mean 0 & variance \( m \beta \).

In particular,
\[
\langle \text{kinetic energy} \rangle_p = \frac{1}{2} N \cdot \beta \cdot kT
\]

So for particle it is \( \frac{1}{2} \beta kT \)
per degree of freedom it is \( \frac{1}{2} kT \)

(does not depend on \( m \) ?)

The ideal gas:

\[ Z = \left( \frac{2\pi m}{\beta} \right)^{N/2} \int e^{-\beta u/V(\mathbf{p})} \frac{d\mathbf{p}}{N!} \]

If the particles are non-interacting
\[ \frac{1}{N} \log Z \approx \frac{1}{N} \log \left( \frac{V(\mathbf{p})}{N!} \right) \]

Easy to see if we consider a sequence \( N \) of boxes and a sequence \( N \) of particle numbers

\[ N! \approx e^{-N} N^N \]

Thermodynamics: The study of macroscopic matter.

Recommended reference: Fermi

The thermal equilibrium states of a given system can
be described by giving a small number of parameters. Example: A sample of gas is fully described by its pressure and temperature.

Point of view: The thermal equilibrium states of a given system form an abstract space, like a manifold — that is, it doesn’t come with a preferred parametrization.

By convention we abstract away the overall quantity of matter involved (yet the proportions of different ingredients are kept)