

starting idea in statistical mechanics:
 Instead of looking at individual phase points,
 study probability measures on phase space; replace
 physical quantities by their means.

Boltzmann/Gibbs: The appropriate measure
 has density prop. to $e^{-\beta H(x)}$.
 This measure will be invariant under the physics flow.
 $\beta = \frac{1}{kT}$, where T is the absolute temperature
 & $k =$ Boltzmann's constant =

Average kinetic energy per particle = $\frac{d}{2} kT$
 This is "the canonical ensemble".

Notation $x = (\vec{q}_1, \dots, \vec{p}_N)$

Probability measure: $C \cdot e^{-\beta H(x)} \frac{d\vec{q}_1 \dots d\vec{p}_N}{N!}$

Conventionally, $C = \frac{1}{Z}$ where $Z =$ "the canonical partition function"

$$Z = \int e^{-\beta H(x)} dx$$

The equilibrium value of the energy (as a function
 of β or T)

$$\begin{aligned} \langle H \rangle_\beta &:= \frac{1}{Z} \int H e^{-\beta H(x)} dx \\ &= \frac{1}{Z} \left(-\frac{d}{d\beta} \right) \int e^{-\beta H(x)} dx = -\frac{d}{d\beta} (\log Z) \end{aligned}$$

$$= \frac{1}{Z} \left(-\frac{d}{d\beta} \right) \int e^{-\beta H(x)} dx = -\frac{d}{d\beta} (\log Z)$$

Integrals over p are more or less trivial, as they are Gaussians. So

$$Z = \left(\frac{2\pi m}{\beta} \right)^{Nd/2} \int e^{-\beta V(q)} \frac{dq}{N!} \quad \left(m \text{ is the mass of a particle} \right)$$

Let p_{ij} be the j 'th component of \vec{p}_i . Then $\{p_{ij}\}$ are independent Gaussian random variables with mean 0 & variance mKT .

In particular,

$$\langle \text{kinetic energy} \rangle_{\beta} = \frac{1}{2} N \cdot d \cdot KT$$

So per particle it is $\frac{1}{2} dKT$

per degree of freedom it is $\frac{1}{2} KT$

(does not depend on m !)

The ideal gas:

The chalkboard contains the following content:

- Equation: $Z = \left(\frac{2\pi m}{\beta} \right)^{Nd/2} \int e^{-\beta V(q)} \frac{dq}{N!}$
- Equation: $Z \approx \left[\left(\frac{2\pi m}{\beta} \right)^{d/2} c \frac{V(\Lambda)}{N} \right]^N$ where $c = \rho^{-d}$ and $\rho = \text{density}$.
- Equation: $\frac{1}{N} \log Z \approx \log \left[\left(\frac{2\pi m}{\beta} \right)^{d/2} \left(\frac{c}{\rho} \right) \right]$
- Text: "Easy to see. If we consider a sequence Λ_n of boxes, and a sequence N_n of particle numbers"
- Text: "if the particles are non-interacting integral easy."
- Equation: $Z = \left(\frac{2\pi m}{\beta} \right)^{\frac{Nd}{2}} \frac{(V(\Lambda))^N}{N!}$
- Equation: $= \frac{1}{N!} \left[\left(\frac{2\pi m}{\beta} \right)^{d/2} V(\Lambda) \right]^N$
- Equation: $N! \approx e^{-N} N^N$
- Text: "with $V(\Lambda_n) \rightarrow \infty$ "
- Equation: $\frac{N_n}{V(\Lambda_n)} \rightarrow \rho \in (0, \infty)$
- Text: "then $\frac{1}{N_n} \log Z_n \rightarrow \frac{d}{2} \log \left(\frac{2\pi m}{\beta} \right) - \log \rho + 1$ "
- Text: "thermodynamic limit"

Thermodynamics: The study of macroscopic matter;

Recommended reference: Fermi

The thermal equilibrium states of a given system can

be described by giving a small number of parameters.

Example: A sample of gas is fully described by its pressure and temperature.

Point of view: The thermal equilibrium states of a given system form an abstract space, like a manifold — that is, it doesn't come with a preferred parametrization.

By convention we abstract away the overall quantity of matter involved (yet the proportions of different ingredients are kept)