

# Bonn Talk

July-29-09  
5:36 AM

## Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots

Dear Bar-Natan, Bonn August 2009, <http://www.math.toronto.edu/~drorbn/Talks/Bonn-0908>  
Convolutions statement (Kashiwara-Vergne). Convolutions of invariant functions on a Lie group agree with convolutions of invariant functions on its Lie algebra. More accurately, let  $G$  be a finite dimensional Lie group and let  $\mathfrak{g}$  be its Lie algebra, let  $j : \mathfrak{g} \rightarrow \mathbb{R}$  be the Jacobian of the exponential map  $\exp : \mathfrak{g} \rightarrow G$ , and let  $\Phi : \text{Fun}(G) \rightarrow \text{Fun}(\mathfrak{g})$  be given by  $\Phi(f)(x) := j^{-1}f(\exp x)$ . Then if  $f, g \in \text{Fun}(G)$  are Ad-invariant and supported near the identity, then

$$\Phi(f) * \Phi(g) = \Phi(f * g).$$

Group-Algebra statement. There exists  $\omega \in \text{Fun}(\mathfrak{g})^{\otimes 2}$  so that for every  $\phi, \psi \in \text{Fun}(\mathfrak{g})^{\otimes 2}$  (with small support), the following holds in  $\mathcal{U}(\mathfrak{g})$ :

$$\int\int \phi(x)\psi(y) \omega_{xy} e^{xy} = \int\int \phi(x)\psi(y) \omega_{xy} e^{xy}, \quad (\text{whh, this is Duflo})$$

Unitary statement. There exists  $\omega \in \text{Fun}(\mathfrak{g})^{\otimes 2}$  (an infinite order) tangential differential operator  $V$  defined on  $\text{Fun}(\mathfrak{g}_x \times \mathfrak{g}_y)$  so that

$$(1) V e^{xy} = e^x e^y V, \quad (\text{allowing } \mathcal{U}(\mathfrak{g})\text{-valued functions})$$

$$(2) VV^* = I, \quad (3) V_{\omega(x,y)} = \omega_{xy} V_y$$

Algebraic statement. With  $I\mathfrak{g} := \mathfrak{g}^* \rtimes \mathfrak{g}$ , with  $c : \mathcal{U}(I\mathfrak{g}) \rightarrow \mathcal{U}(\mathfrak{g})/\mathcal{U}(\mathfrak{g}) = \mathcal{S}(\mathfrak{g}^*)$  the obvious projection, with  $S$  the transpose of  $I\mathfrak{g}$ , with  $W$  the automorphism of  $\mathcal{U}(\mathfrak{g})$  induced by flipping the sign of  $\mathfrak{g}^*$ , with  $r \in \mathfrak{g}^* \otimes \mathfrak{g}$  the identity element and with  $R = e^r \in \mathcal{U}(\mathfrak{g}) \otimes \mathcal{U}(\mathfrak{g})$  there exist  $\omega \in \mathcal{S}(\mathfrak{g}^*)$  and  $V \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$  so that

$$(1) V(\Delta \otimes 1)(R) = R^{10} R^{23} V \text{ in } \mathcal{U}(I\mathfrak{g})^{\otimes 2} \otimes \mathcal{U}(\mathfrak{g})$$

$$(2) V \cdot SWV = 1, \quad (3) (c \otimes c)(V \Delta(c)) = \omega \otimes \omega$$

Diagrammatic statement. Let  $R = \exp \frac{x}{2} \in \mathcal{A}^*(\mathbb{T})$ . There exist  $\omega \in \mathcal{A}^*(\mathbb{T})$  and  $V \in \mathcal{A}^*(\mathbb{T})$  so that:

$$(1) \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Knot-Theoretic statement. There exists a homomorphic expansion  $Z$  for trivalent w-tangles. In particular,  $Z$  should respect R4 and intertwine annulus and disk upzips:

$$(1) \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

(2)  $\text{Diagram 1} = \text{Diagram 2}$

(3)  $\text{Diagram 1} = \text{Diagram 2}$

(4)  $\text{Diagram 1} = \text{Diagram 2}$

(5)  $\text{Diagram 1} = \text{Diagram 2}$

(6)  $\text{Diagram 1} = \text{Diagram 2}$

(7)  $\text{Diagram 1} = \text{Diagram 2}$

(8)  $\text{Diagram 1} = \text{Diagram 2}$

(9)  $\text{Diagram 1} = \text{Diagram 2}$

(10)  $\text{Diagram 1} = \text{Diagram 2}$

(11)  $\text{Diagram 1} = \text{Diagram 2}$

(12)  $\text{Diagram 1} = \text{Diagram 2}$

(13)  $\text{Diagram 1} = \text{Diagram 2}$

(14)  $\text{Diagram 1} = \text{Diagram 2}$

(15)  $\text{Diagram 1} = \text{Diagram 2}$

(16)  $\text{Diagram 1} = \text{Diagram 2}$

(17)  $\text{Diagram 1} = \text{Diagram 2}$

(18)  $\text{Diagram 1} = \text{Diagram 2}$

(19)  $\text{Diagram 1} = \text{Diagram 2}$

(20)  $\text{Diagram 1} = \text{Diagram 2}$

(21)  $\text{Diagram 1} = \text{Diagram 2}$

(22)  $\text{Diagram 1} = \text{Diagram 2}$

(23)  $\text{Diagram 1} = \text{Diagram 2}$

(24)  $\text{Diagram 1} = \text{Diagram 2}$

(25)  $\text{Diagram 1} = \text{Diagram 2}$

(26)  $\text{Diagram 1} = \text{Diagram 2}$

(27)  $\text{Diagram 1} = \text{Diagram 2}$

(28)  $\text{Diagram 1} = \text{Diagram 2}$

(29)  $\text{Diagram 1} = \text{Diagram 2}$

(30)  $\text{Diagram 1} = \text{Diagram 2}$

(31)  $\text{Diagram 1} = \text{Diagram 2}$

(32)  $\text{Diagram 1} = \text{Diagram 2}$

(33)  $\text{Diagram 1} = \text{Diagram 2}$

(34)  $\text{Diagram 1} = \text{Diagram 2}$

(35)  $\text{Diagram 1} = \text{Diagram 2}$

(36)  $\text{Diagram 1} = \text{Diagram 2}$

(37)  $\text{Diagram 1} = \text{Diagram 2}$

(38)  $\text{Diagram 1} = \text{Diagram 2}$

(39)  $\text{Diagram 1} = \text{Diagram 2}$

(40)  $\text{Diagram 1} = \text{Diagram 2}$

(41)  $\text{Diagram 1} = \text{Diagram 2}$

(42)  $\text{Diagram 1} = \text{Diagram 2}$

(43)  $\text{Diagram 1} = \text{Diagram 2}$

(44)  $\text{Diagram 1} = \text{Diagram 2}$

(45)  $\text{Diagram 1} = \text{Diagram 2}$

(46)  $\text{Diagram 1} = \text{Diagram 2}$

(47)  $\text{Diagram 1} = \text{Diagram 2}$

(48)  $\text{Diagram 1} = \text{Diagram 2}$

(49)  $\text{Diagram 1} = \text{Diagram 2}$

(50)  $\text{Diagram 1} = \text{Diagram 2}$

(51)  $\text{Diagram 1} = \text{Diagram 2}$

(52)  $\text{Diagram 1} = \text{Diagram 2}$

(53)  $\text{Diagram 1} = \text{Diagram 2}$

(54)  $\text{Diagram 1} = \text{Diagram 2}$

(55)  $\text{Diagram 1} = \text{Diagram 2}$

(56)  $\text{Diagram 1} = \text{Diagram 2}$

(57)  $\text{Diagram 1} = \text{Diagram 2}$

(58)  $\text{Diagram 1} = \text{Diagram 2}$

(59)  $\text{Diagram 1} = \text{Diagram 2}$

(60)  $\text{Diagram 1} = \text{Diagram 2}$

(61)  $\text{Diagram 1} = \text{Diagram 2}$

(62)  $\text{Diagram 1} = \text{Diagram 2}$

(63)  $\text{Diagram 1} = \text{Diagram 2}$

(64)  $\text{Diagram 1} = \text{Diagram 2}$

(65)  $\text{Diagram 1} = \text{Diagram 2}$

(66)  $\text{Diagram 1} = \text{Diagram 2}$

(67)  $\text{Diagram 1} = \text{Diagram 2}$

(68)  $\text{Diagram 1} = \text{Diagram 2}$

(69)  $\text{Diagram 1} = \text{Diagram 2}$

(70)  $\text{Diagram 1} = \text{Diagram 2}$

(71)  $\text{Diagram 1} = \text{Diagram 2}$

(72)  $\text{Diagram 1} = \text{Diagram 2}$

(73)  $\text{Diagram 1} = \text{Diagram 2}$

(74)  $\text{Diagram 1} = \text{Diagram 2}$

(75)  $\text{Diagram 1} = \text{Diagram 2}$

(76)  $\text{Diagram 1} = \text{Diagram 2}$

(77)  $\text{Diagram 1} = \text{Diagram 2}$

(78)  $\text{Diagram 1} = \text{Diagram 2}$

(79)  $\text{Diagram 1} = \text{Diagram 2}$

(80)  $\text{Diagram 1} = \text{Diagram 2}$

(81)  $\text{Diagram 1} = \text{Diagram 2}$

(82)  $\text{Diagram 1} = \text{Diagram 2}$

(83)  $\text{Diagram 1} = \text{Diagram 2}$

(84)  $\text{Diagram 1} = \text{Diagram 2}$

(85)  $\text{Diagram 1} = \text{Diagram 2}$

(86)  $\text{Diagram 1} = \text{Diagram 2}$

(87)  $\text{Diagram 1} = \text{Diagram 2}$

(88)  $\text{Diagram 1} = \text{Diagram 2}$

(89)  $\text{Diagram 1} = \text{Diagram 2}$

(90)  $\text{Diagram 1} = \text{Diagram 2}$

(91)  $\text{Diagram 1} = \text{Diagram 2}$

(92)  $\text{Diagram 1} = \text{Diagram 2}$

(93)  $\text{Diagram 1} = \text{Diagram 2}$

(94)  $\text{Diagram 1} = \text{Diagram 2}$

(95)  $\text{Diagram 1} = \text{Diagram 2}$

(96)  $\text{Diagram 1} = \text{Diagram 2}$

(97)  $\text{Diagram 1} = \text{Diagram 2}$

(98)  $\text{Diagram 1} = \text{Diagram 2}$

(99)  $\text{Diagram 1} = \text{Diagram 2}$

(100)  $\text{Diagram 1} = \text{Diagram 2}$

(101)  $\text{Diagram 1} = \text{Diagram 2}$

(102)  $\text{Diagram 1} = \text{Diagram 2}$

(103)  $\text{Diagram 1} = \text{Diagram 2}$

(104)  $\text{Diagram 1} = \text{Diagram 2}$

(105)  $\text{Diagram 1} = \text{Diagram 2}$

(106)  $\text{Diagram 1} = \text{Diagram 2}$

(107)  $\text{Diagram 1} = \text{Diagram 2}$

(108)  $\text{Diagram 1} = \text{Diagram 2}$

(109)  $\text{Diagram 1} = \text{Diagram 2}$

(110)  $\text{Diagram 1} = \text{Diagram 2}$

(111)  $\text{Diagram 1} = \text{Diagram 2}$

(112)  $\text{Diagram 1} = \text{Diagram 2}$

(113)  $\text{Diagram 1} = \text{Diagram 2}$

(114)  $\text{Diagram 1} = \text{Diagram 2}$

(115)  $\text{Diagram 1} = \text{Diagram 2}$

(116)  $\text{Diagram 1} = \text{Diagram 2}$

(117)  $\text{Diagram 1} = \text{Diagram 2}$

(118)  $\text{Diagram 1} = \text{Diagram 2}$

(119)  $\text{Diagram 1} = \text{Diagram 2}$

(120)  $\text{Diagram 1} = \text{Diagram 2}$

(121)  $\text{Diagram 1} = \text{Diagram 2}$

(122)  $\text{Diagram 1} = \text{Diagram 2}$

(123)  $\text{Diagram 1} = \text{Diagram 2}$

(124)  $\text{Diagram 1} = \text{Diagram 2}$

(125)  $\text{Diagram 1} = \text{Diagram 2}$

(126)  $\text{Diagram 1} = \text{Diagram 2}$

(127)  $\text{Diagram 1} = \text{Diagram 2}$

(128)  $\text{Diagram 1} = \text{Diagram 2}$

(129)  $\text{Diagram 1} = \text{Diagram 2}$

(130)  $\text{Diagram 1} = \text{Diagram 2}$

(131)  $\text{Diagram 1} = \text{Diagram 2}$

(132)  $\text{Diagram 1} = \text{Diagram 2}$

(133)  $\text{Diagram 1} = \text{Diagram 2}$

(134)  $\text{Diagram 1} = \text{Diagram 2}$

(135)  $\text{Diagram 1} = \text{Diagram 2}$

(136)  $\text{Diagram 1} = \text{Diagram 2}$

(137)  $\text{Diagram 1} = \text{Diagram 2}$

(138)  $\text{Diagram 1} = \text{Diagram 2}$

(139)  $\text{Diagram 1} = \text{Diagram 2}$

(140)  $\text{Diagram 1} = \text{Diagram 2}$

(141)  $\text{Diagram 1} = \text{Diagram 2}$

(142)  $\text{Diagram 1} = \text{Diagram 2}$

(143)  $\text{Diagram 1} = \text{Diagram 2}$

(144)  $\text{Diagram 1} = \text{Diagram 2}$

(145)  $\text{Diagram 1} = \text{Diagram 2}$

(146)  $\text{Diagram 1} = \text{Diagram 2}$

(147)  $\text{Diagram 1} = \text{Diagram 2}$

(148)  $\text{Diagram 1} = \text{Diagram 2}$

(149)  $\text{Diagram 1} = \text{Diagram 2}$

(150)  $\text{Diagram 1} = \text{Diagram 2}$

(151)  $\text{Diagram 1} = \text{Diagram 2}$

(152)  $\text{Diagram 1} = \text{Diagram 2}$

(153)  $\text{Diagram 1} = \text{Diagram 2}$

(154)  $\text{Diagram 1} = \text{Diagram 2}$

(155)  $\text{Diagram 1} = \text{Diagram 2}$

(156)  $\text{Diagram 1} = \text{Diagram 2}$

(157)  $\text{Diagram 1} = \text{Diagram 2}$

(158)  $\text{Diagram 1} = \text{Diagram 2}$

(159)  $\text{Diagram 1} = \text{Diagram 2}$

(160)  $\text{Diagram 1} = \text{Diagram 2}$

(161)  $\text{Diagram 1} = \text{Diagram 2}$

(162)  $\text{Diagram 1} = \text{Diagram 2}$

(163)  $\text{Diagram 1} = \text{Diagram 2}$

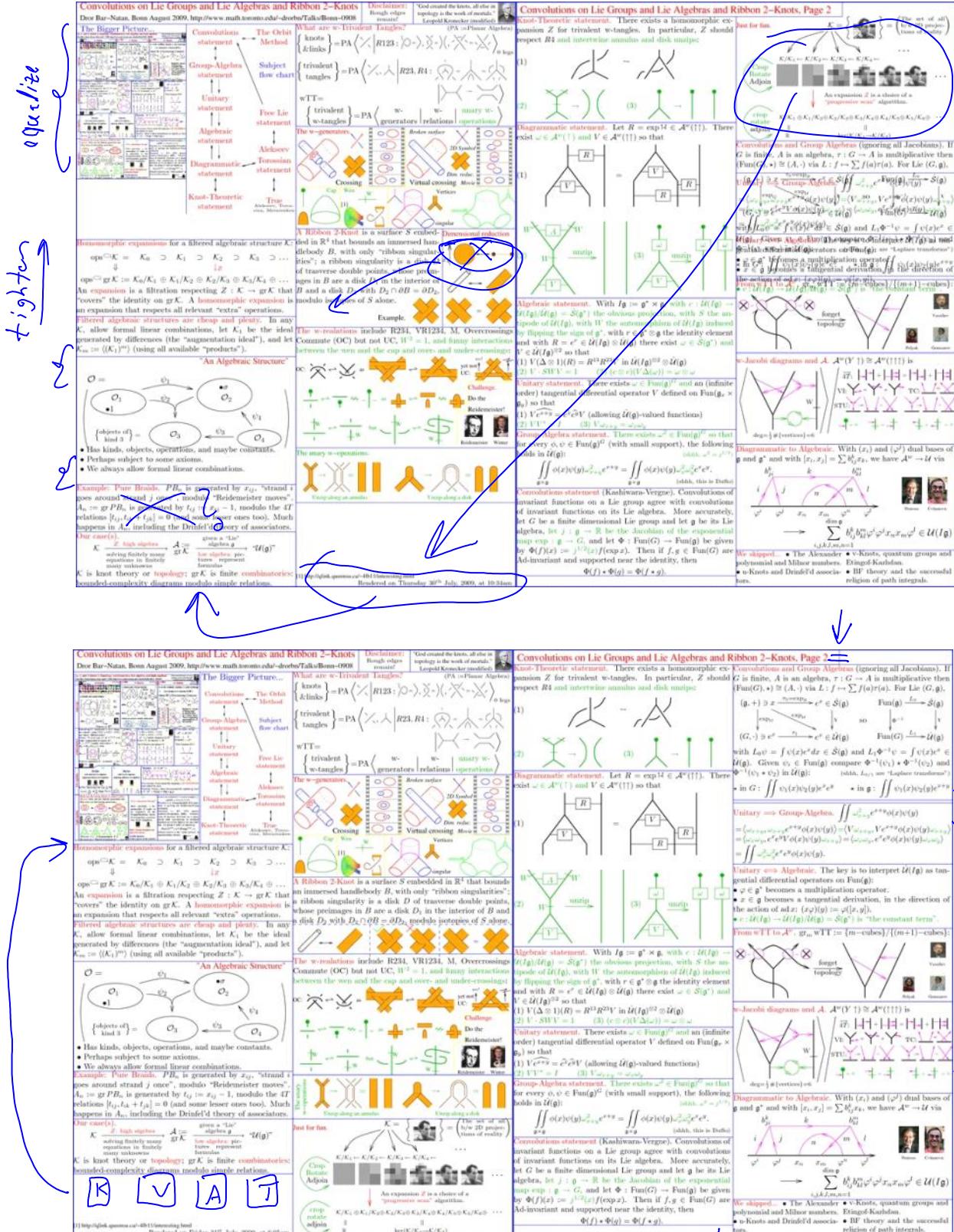
(164)  $\text{Diagram 1} = \text{Diagram 2}$

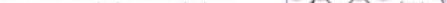
(165)  $\text{Diagram 1} = \text{Diagram 2}$

(166)  $\text{Diagram 1} = \text{Diagram 2}$

(167)  $\text{Diagram 1} = \text{Diagram 2}$

(168)  $\text{Diagram 1} = \text{Diagram 2}$



Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots	Disclaimer: Rough edges God created the knots, all else is topology is the work of mortals. Legendre-Konigsberg (1750)	Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots, Page 2
Drew Bar-Natan, Brown August 2009, <a href="http://www.math.toronto.edu/~dbran/Talks/Brown-0908/">http://www.math.toronto.edu/~dbran/Talks/Brown-0908/</a>	 Knots are Trivalent Tangles! Rough edges Legendre-Konigsberg (1750) HOMFLY (Lie Algebra)	Knot-Theoretic statement: There exists a homomorphic extension $Z$ for trivalent w-tangles. In particular, $Z$ should respect $H4$ and intertwine annulus and disk maps:  From wTT to $A^{\otimes *}$ , $\text{gr}_{\text{wt}}$ , $\text{wTT} := \{m\text{-cubes}\}/\{(m+1)\text{-cubes}\}$ .

**Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots, Page 2**

**Knot-Theoretic statement.** There exists a homomorphic expansion  $Z$  for trivial  $w$ -tangles. In particular,  $Z$  should respect  $R4$  and intertwine annulus and disk sumps:

(1)   
(2)   
(3)   
(4)   
(5)   
(6)   
(7)   
(8)   
(9)   
(10)   
(11)   
(12)   
(13)   
(14)   
(15)   
(16)   
(17)   
(18)   
(19)   
(20)   
(21)   
(22)   
(23)   
(24)   
(25)   
(26)   
(27)   
(28)   
(29)   
(30)   
(31)   
(32)   
(33)   
(34)   
(35)   
(36)   
(37)   
(38)   
(39)   
(40)   
(41)   
(42)   
(43)   
(44)   
(45)   
(46)   
(47)   
(48)   
(49)   
(50)   
(51)   
(52)   
(53)   
(54)   
(55)   
(56)   
(57)   
(58)   
(59)   
(60)   
(61)   
(62)   
(63)   
(64)   
(65)   
(66)   
(67)   
(68)   
(69)   
(70)   
(71)   
(72)   
(73)   
(74)   
(75)   
(76)   
(77)   
(78)   
(79)   
(80)   
(81)   
(82)   
(83)   
(84)   
(85)   
(86)   
(87)   
(88)   
(89)   
(90)   
(91)   
(92)   
(93)   
(94)   
(95)   
(96)   
(97)   
(98)   
(99)   
(100)   
(101)   
(102)   
(103)   
(104)   
(105)   
(106)   
(107)   
(108)   
(109)   
(110)   
(111)   
(112)   
(113)   
(114)   
(115)   
(116)   
(117)   
(118)   
(119)   
(120)   
(121)   
(122)   
(123)   
(124)   
(125)   
(126)   
(127)   
(128)   
(129)   
(130)   
(131)   
(132)   
(133)   
(134)   
(135)   
(136)   
(137)   
(138)   
(139)   
(140)   
(141)   
(142)   
(143)   
(144)   
(145)   
(146)   
(147)   
(148)   
(149)   
(150)   
(151)   
(152)   
(153)   
(154)   
(155)   
(156)   
(157)   
(158)   
(159)   
(160)   
(161)   
(162)   
(163)   
(164)   
(165)   
(166)   
(167)   
(168)   
(169)   
(170)   
(171)   
(172)   
(173)   
(174)   
(175)   
(176)   
(177)   
(178)   
(179)   
(180)   
(181)   
(182)   
(183)   
(184)   
(185)   
(186)   
(187)   
(188)   
(189)   
(190)   
(191)   
(192)   
(193)   
(194)   
(195)   
(196)   
(197)   
(198)   
(199)   
(200)   
(201)   
(202)   
(203)   
(204)   
(205)   
(206)   
(207)   
(208)   
(209)   
(210)   
(211)   
(212)   
(213)   
(214)   
(215)   
(216)   
(217)   
(218)   
(219)   
(220)   
(221)   
(222)   
(223)   
(224)   
(225)   
(226)   
(227)   
(228)   
(229)   
(230)   
(231)   
(232)   
(233)   
(234)   
(235)   
(236)   
(237)   
(238)   
(239)   
(240)   
(241)   
(242)   
(243)   
(244)   
(245)   
(246)   
(247)   
(248)   
(249)   
(250)   
(251)   
(252)   
(253)   
(254)   
(255)   
(256)   
(257)   
(258)   
(259)   
(260)   
(261)   
(262)   
(263)   
(264)   
(265)   
(266)   
(267)   
(268)   
(269)   
(270)   
(271)   
(272)   
(273)   
(274)   
(275)   
(276)   
(277)   
(278)   
(279)   
(280)   
(281)   
(282)   
(283)   
(284)   
(285)   
(286)   
(287)   
(288)   
(289)   
(290)   
(291)   
(292)   
(293)   
(294)   
(295)   
(296)   
(297)   
(298)   
(299)   
(300)   
(301)   
(302)   
(303)   
(304)   
(305)   
(306)   
(307)   
(308)   
(309)   
(310)   
(311)   
(312)   
(313)   
(314)   
(315)   
(316)   
(317)   
(318)   
(319)   
(320)   
(321)   
(322)   
(323)   
(324)   
(325)   
(326) <img alt="Diagram showing a trefoil knot with a

\* The simplest problem  
Hyperbolic geometry solves.