The 2D Lie Algebra on Arbitrary Arrow Diagrams

July-07-09
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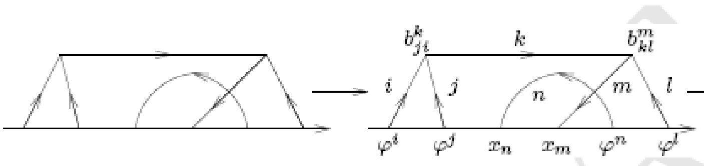
First, a quote from the current state of WKO:



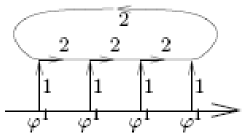
$$\longrightarrow \sum_{i,j,k,l,m,n=1}^{\dim\mathfrak{g}} b^k_{ij} b^m_{kl} \varphi^i \varphi^j x_n x_m \varphi^l \in \mathcal{U}(I\mathfrak{g}) \qquad (17)$$

3.6.3. *Example: The 2 Dimensional Non-Abelian Lie Algebra.* Let $\mathfrak{g}$ be the Lie algebra with two generators $x_{1,2}$ satisfying $[x_1, x_2] = x_2$, so that the only non-vanishing structure constants b^k_{ij} of $\mathfrak{g}$ are $b^2_{12} = -b^2_{21} = 1$. Let $\varphi^i \in \mathfrak{g}^*$ be the dual basis of x_i; by an easy calculation, we find that in $I\mathfrak{g}$ the element φ^1 is central, while $[x_1, \varphi^2] = -\varphi^2$ and $[x_2, \varphi^2] = \varphi^1$. We calculate $\mathcal{T}^w_\mathfrak{g}(D_L)$, $\mathcal{T}^w_\mathfrak{g}(D_R)$ and $\mathcal{T}^w_\mathfrak{g}(w_k)$ using the "in basis" technique of Equation (17). The outputs of these calculations lie in $\mathcal{U}(I\mathfrak{g})$; we display these results in a PBW basis in which the elements of $\mathfrak{g}^*$ precede the elements of $\mathfrak{g}$:

$$\mathcal{T}^w_\mathfrak{g}(D_L) = x_1\varphi^1 + x_2\varphi^2 = \varphi^1 x_1 + \varphi^2 x_2 + [x_2, \varphi^2] = \varphi^1 x_1 + \varphi^2 x_2 + \varphi_1, \qquad (18)$$

$$\mathcal{T}^w_\mathfrak{g}(D_R) = \varphi^1 x_1 + \varphi^2 x_2, \qquad (19)$$

$$\mathcal{T}^w_\mathfrak{g}(w_k) = (\varphi^1)^k. \qquad (20)$$



For the last assertion above, note that all non-vanishing structure constants b^k_{ij} in our case have $k = 2$, and therefore all indices corresponding to edges that exit an internal vertex must be set equal to 2. This forces the "hub" of w_k to be marked 2 and therefore the legs to be marked 1, and therefore w_k is mapped to $(\varphi^1)^k$.

Note that the calculations in (18) are consistent with the relation $D_L - D_R = w_1$ of Theorem 3.13 and that they show that other than that relation, the generators of $\mathcal{A}^w$ are linearly independent.

No ideas.