Maxwell's Equations

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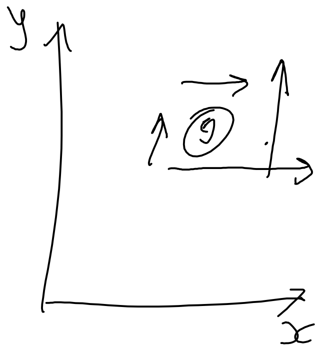
current, mag. field

ρ, j, E, B

charg density, elec. field



ρ: function $\quad \rho: \mathbb{R}^4 \to \mathbb{R}$

$E, B, j: \mathbb{R}^4 \to \mathbb{R}^3$

$$\{f\} \xrightarrow[\nabla]{\text{grad}} \left\{\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}\right\} \xrightarrow[\nabla\times]{\text{curl}} \left\{\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}\right\} \xrightarrow[\nabla\cdot]{\text{div}} \{\mu\}$$

$f: \mathbb{R}^3 \to \mathbb{R}$ $\quad \{g: \mathbb{R}^3 \to \mathbb{R}^3\}$ $\quad \{h: \mathbb{R}^3 \to \mathbb{R}^3\}$

$$f \to \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix}$$

$(x_1\ x_2\ x_3) \leftrightarrow (x, y, z)$

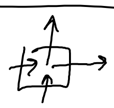
$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \xrightarrow{\text{curl}} \begin{array}{c|c} \begin{pmatrix} \partial_2 f_3 - \partial_3 f_2 \\ \partial_3 f_1 - \partial_1 f_3 \\ \partial_1 f_2 - \partial_2 f_1 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

$\nabla \times f$

$$\det\begin{pmatrix} A_1 & B_1 & i \\ A_2 & B_2 & j \\ A_3 & B_3 & k \end{pmatrix} = \begin{pmatrix} \vdots \\ \\ A_1 B_2 - A_2 B_1 \end{pmatrix}$$

$$\nabla = \begin{pmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{pmatrix} \quad f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$h = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \xrightarrow{\text{div}} \partial_1 h_1 + \partial_2 h_2 + \partial_3 h_3$$



0th Maxwell equation: "continuity" $\exists \sigma \in S_4$

up to constants and signs

cons. of charge $\quad$ 0. $-\operatorname{div} j = \frac{\partial}{\partial t}\rho$

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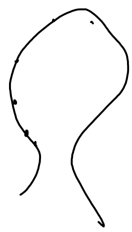
charge creates Electric field. $\sim$1. $\operatorname{div} E = \rho$

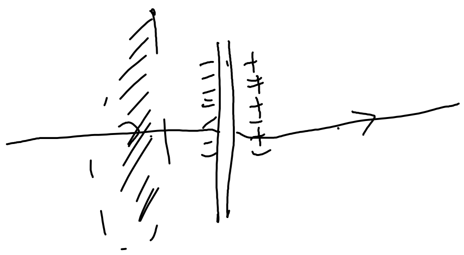
Electromagnets $\sim$2. $\operatorname{curl} B = j + \underbrace{\frac{\partial E}{\partial t}}_{\text{Maxwell term}}$

generators $\sim$3 $\operatorname{curl} E = \frac{\partial B}{\partial t}$

No monopoles $\sim$4 $\operatorname{div} B = 0$



no magnetic monopoles



"The Maxwell term"

5. Motion of a particle in E&B:

$$F = e(E + v \times B)$$

charge