Determinants without division

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(No such thing) (at least, not well behaved)

$$\begin{pmatrix} 1 & & 0 & & * \\ & 1 & & & * \\ 0 & & 1 & & * \\ & & & p & * \\ * & * & * & * & * \end{pmatrix} \xrightarrow{\text{row \& col ops}} \begin{pmatrix} 1 & & 0 & & 0 \\ & 1 & & & 0 \\ 0 & & 1 & & 0 \\ & & & p & A \\ 0 & 0 & 0 & B & C \end{pmatrix} \rightarrow 2\times 2$$

Aside: The 2×2 Case:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 4 \\ 0 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix}$$

$2\cdot 8 - 4\cdot 6 = -8$

gcd of all minors of $A-\lambda I$:

$\begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1-\lambda & 0 \\ 6 & 1-\lambda \end{pmatrix} \rightarrow 1-\lambda$

$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{pmatrix} \rightarrow 1$

Q: Can the full S-equivalence class of a Seifert matrix be computed in polynomial time?