Non Commutative Gaussian Elimination - Program 5

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~dorbn/Misc/SchreierSimsRubik/.

Pensieve Header: NCGE Program 5 - "TeaseFeed" is implemented with lovely results.

The Cube
The Generating Permutations

\[ n = 54; \text{\$RecursionLimit = 2^16; } \]

\[ \text{Generators = \{ } \]

\[ \text{M[\{18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17,} \]

\[ 45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43,} \]

\[ 37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48},} \]

\[ \text{\{BottomFace}, 1\} }, \]

\[ \text{M[\{1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39,} \]

\[ 17, 18, 19, 20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36,} \]

\[ 12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54},} \]

\[ \text{\{TopFace}, 1\} }, \]

\[ \text{M[\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,} \]

\[ 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46,} \]

\[ 39, 42, 45, 38, 41, 44, 37, 40, 43, 30, 29, 28, 49, 50, 51, 52, 53, 54},} \]

\[ \text{\{FrontFace}, 1\} }, \]

\[ \text{M[\{3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14,} \]

\[ 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,} \]

\[ 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16},} \]

\[ \text{\{BackFace}, 1\} }, \]

\[ \text{M[\{13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16,} \]

\[ 17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35,} \]

\[ 36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 4, 50, 51, 7, 53, 54},} \]

\[ \text{\{LeftFace}, 1\} }, \]

\[ \text{M[\{1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18,} \]

\[ 27, 36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34,} \]

\[ 37, 38, 15, 40, 41, 24, 43, 44, 33, 34, 47, 39, 30, 42, 52, 53, 45},} \]

\[ \text{\{RightFace}, 1\} } \];
Program 5

Clear[s, M, T]; TC = 0;
M /: M[a1_, (w1___), m1_] \[DoubleLongEqual] M[a1[[a2]], (w1, w2), m1 + m2];
M /: Inverse[M[a_, w_, m_]] := M[Ordering[a], -Reverse[w], m];
Feed[M[Range[n], _]] := {};
Feed[M[a_, (w___), m_]] := Module[
   {i, j, x, k, l, m1, m2},
   If[Head[x = s[i, j]] == Integer,
     (* then *) If[m \[Equal] T[x][[3]],
       Feed[Inverse[ReplacePart[T[x], {x}, 2]] \[DoubleLongEqual] M[a, {w}, m]],
       T[x][i, j] = ++TC] = M[a, {w}, m];
     Feed[Inverse[M[a, {w}, m]] \[DoubleLongEqual] ReplacePart[T[x], {x}, 2]]
   ],
   (* else *) T[x][i, j] = ++TC] = M[a, {w}, m];
   Do[
     If[Head[y = s[k, l]] == Integer,
       Feed[ReplacePart[T[x], {y}, 2]] \[DoubleLongEqual] T[x][i, j],
       Feed[ReplacePart[T[x][i, j] \[DoubleLongEqual] T[x][i, j]]],
       {k, n}, {l, n}
   ]
   ];

Images[i_] := Prepend[Select[Range[n], Head[s[i, #]] == Integer &], i];
MoveCount[i_, i_] := 0;
MoveCount[i_, j_] := T[s[i, j]][[3]];
TMC[] := Sum[Total[MoveCount[i, #] \[DoubleLongEqual] Images[i]], {i, n}];
Optimize[] := Timing[
   Do[
     If[Head[x = s[i, j]] == Integer, Do[
       If[Head[y = s[k, l]] == Integer,
         Feed[ReplacePart[T[x], {y}, 2]] \[DoubleLongEqual] T[x][i, j],
         Feed[ReplacePart[T[x][i, j] \[DoubleLongEqual] T[x][i, j]]],
         {k, n}, {l, n}]
       ],
     ],
   ];
   gens = Rest[Generators];
g = 0;
Print[Timing[
   (++g; Feed[##] \[DoubleLongEqual] Product[Length[Images[i]], {i, n}] \[DoubleLongEqual] Join[gens, Inverse /@ gens]]);
   Print[tmc = TMC[]];
While[
   Last[opt = Optimize[]] \[NotEqual] tmc,
   tmc = Last[opt];
   Print[opt]
]
The Worst Case Scenario

\[
\text{Sum}[\text{Max}[\text{MoveCount}[i, \#] \& /\@ \text{Images}[i]], \{i, n\}]
\]

206

And finally - one cube really solved...

\[
P /: P[a___] ** P[a2___] := P[a][[a2]];
P /: \text{Inverse}[P[a___]] := P @@ \text{Ordering}[[a]];
\text{TeaseFeed}[P @@ \text{Range}[n]] := \{\};
\text{TeaseFeed}[p_P] := \text{Module}[
  \{i, j, sij\},
  \text{For}[i = 1, p[[i]] = i, ++i; j = p[[i]];
  \text{If}[\text{Head}[sij = s[i, j]] == \text{Integer},
    \text{Prepend}[\text{TeaseFeed}[\text{Inverse}[P @@ T[sij][[1]]] ** p], sij],
    \{\}
  \}
]
\]

\[
P @@ \text{Range}[54]
\]

\[
P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
  19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
  37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]
\]

\[
\text{TeaseFeed}[P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
  17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35,
  36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]]
\]

\[
\{}
\]

\[
\text{TeaseFeed}[P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
  17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35,
  36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]]
\]

\[
\{}
\]

\[
\text{gen1} = P @@ \text{Generators}[[1, 1]]
\]

\[
P[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17,
  45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43,
  37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48]
\]
TeaseFeed[gen1]
{5562, 5681, 5820, 5498, 6042, 5504, 6093, 6195, 6177, 6150, 6209, 6224, 6169}

TeaseFeed[gen1] / . t_Integer :> T[t][[2]]
{{4622, 4994}, {-2569, 1, 5066}, {-5066, 2, 5372}, {2538, 4985},
 {4, 5856}, {-5, -4622, 2538, 4622}, {-5709, 4981, 5698}, {4, 6082},
 {-6018, 5, 4622}, {-4, -4983, 6146, 6079}, {-4981, -6018, 6162, 4622},
 {-6168, -6213, -4622, 6213, 4622}, {-4981, -6053, -6123, -6131, 6091, 4622}}

TeaseFeed[gen1] / . t_Integer :> T[t][[2]] / . {
    t_Integer /; t > 0 :> T[t][[2]],
    t_Integer /; t < 0 :> -Reverse[T[-t][[2]]]
}

{{[RightFace], {4982, 2569}}, {[LeftFace], {TopFace}, {2569, 4982}},
 {[4982, -2569], [1, 1], {4985, 966}}, {[966, 1, 966], -RightFace},
 {{FrontFace}, {-5684, 5221, 1}}, {{-1, -4, 1}, -RightFace, {-966, 1, 966}, {RightFace}},
 {{-5373, -1}, -FrontFace, {-4622, 1, 4622}}, {[FrontFace], {-4985, -4981, 4985, 5763}},
 {{-4622, -4}, {-1, 4, 1}, {RightFace}}, {LeftFace}, [1, 6026], {4981, 4983}},
 {{FrontFace}, [-4622, -4], -6092, 6081, 4980}, {RightFace},
 {{-6018, -6091, 4622, 4}, -6146, -4981}, {RightFace, {4981, 6146}, {RightFace}},
 {{FrontFace}, [-6052, -4], -6085, -4985, 6014}, {5691, -6224}, [-1, 4981, 1], {RightFace]]

FixedPoint
Flatten[Replace[#1, {
   t_Integer /; t > 0 :> T[t][[2]],
   t_Integer /; t < 0 :> -Reverse[T[-t][[2]]]
}, {1}] &,
TeaseFeed[gen1]
]

{RightFace, -BackFace, LeftFace, -LeftFace, TopFace, LeftFace, -BackFace, BackFace,
 -LeftFace, TopFace, TopFace, -RightFace, BackFace, -BackFace, TopFace, BackFace,
 -RightFace, FrontFace, LeftFace, -TopFace, -LeftFace, -BackFace, BackFace, -LeftFace, TopFace, TopFace, -RightFace, BackFace, -BackFace, TopFace, BackFace,
 RightFace, BackFace, RightFace, -BackFace, -FrontFace, -RightFace, -FrontFace, RightFace, TopFace, RightFace, FrontFace, RightFace, FrontFace, -RightFace, TopFace, BackFace, RightFace, -BackFace, -TopFace, RightFace, -BackFace, -FrontFace, RightFace, TopFace, RightFace, FrontFace, LeftFace, TopFace, -TopFace, FrontFace, -FrontFace, -LeftFace, FrontFace, -RightFace, -TopFace, FrontFace, RightFace, FrontFace, -TopFace, FrontFace, -RightFace, -TopFace, FrontFace, RightFace, FrontFace, -TopFace, RightFace, FrontFace, -TopFace, TopFace, BackFace, RightFace, -BackFace, -TopFace, RightFace, -BackFace, -FrontFace, RightFace, TopFace, RightFace, FrontFace, LeftFace, TopFace, FrontFace, -TopFace, FrontFace, -LeftFace, FrontFace, -RightFace, -TopFace, FrontFace, RightFace, FrontFace, -TopFace, RightFace, TopFace, FrontFace, -RightFace, -TopFace, RightFace, -RightFace, -FrontFace, TopFace, FrontFace, -TopFace, RightFace, FrontFace, -TopFace, TopFace, -TopFace, -TopFace, -FrontFace, TopFace, TopFace, RightFace}
TeaseFeed[gen1] //. {
  \[x\]_\[\]_ \[\] \rightarrow \[\]_\[\]
  t_\text{Integer} /; t > 0 \[\] \rightarrow T\[t\][[2]],
  t_\text{Integer} /; t < 0 \[\] \rightarrow \text{Reverse}[T\[-t\][[2]]]
}

Flatten[
TeaseFeed[gen1] //. {
  \[x\]_\[\]_ \[\] \rightarrow \[\]_\[\]
  t_\text{Integer} /; t > 0 \[\] \rightarrow T\[t\][[2]],
  t_\text{Integer} /; t < 0 \[\] \rightarrow \text{Reverse}[T\[-t\][[2]]]
}

{RightFace, -BackFace, LeftFace, -LeftFace, TopFace, LeftFace, -BackFace, BackFace, -LeftFace, TopFace, BackFace, -BackFace, TopFace, BackFace, -RightFace, FrontFace, LeftFace, -LeftFace, -BackFace, LeftFace, -LeftFace, -RightFace, TopFace, -TopFace, FrontFace, -RightFace, FrontFace, FrontFace, RightFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, FrontFace, RightPhase}
\begin{verbatim}
cool = Flatten[
  TeaseFeed[gen1] //. {
    -x_ -> -x,
    _integer /; t > 0 -> T[t][[2]],
    _integer /; t < 0 -> Reverse[T[-t][[2]]]
}
] //. {
  {lft___, x_, -x_, rgt___} -> {lft, rgt},
  {lft___, -x_, x_, rgt___} -> {lft, rgt},
  {lft___, x_, -x_, rgt___} -> {lft, -x, rgt},
  {lft___, -x_, -x_, rgt___} -> {lft, x, rgt}
}

{RightFace, -BackFace, -TopFace, -RightFace, TopFace, BackFace, -RightFace, FrontFace, 
LeftFace, -TopFace, -LeftFace, -BackFace, -LeftFace, BackFace, -FrontFace, TopFace, 
-RightFace, -BackFace, TopFace, BackFace, RightFace, RightFace, RightFace, -BackFace, 
-TopFace, -FrontFace, -RightFace, TopFace, RightFace, FrontFace, RightFace, FrontFace, 
-RightFace, TopFace, BackFace, -RightFace, -BackFace, -TopFace, -RightFace, 
-FrontFace, -TopFace, FrontFace, TopFace, RightFace, -FrontFace, LeftFace, TopFace, 
FrontFace, -TopFace, -FrontFace, FrontFace, -RightFace, -FrontFace, 
TopFace, FrontFace, RightFace, -FrontFace, -RightFace, -TopFace, -FrontFace, 
FrontFace, RightFace, -TopFace, -TopFace, BackFace, RightFace, -BackFace, -RightFace, 
-TopFace, RightFace, -TopFace, -FrontFace, -FrontFace, RightFace, FrontFace, 
RightFace, RightFace, -TopFace, -TopFace, -TopFace, TopFace, RightFace, TopFace, RightFace, 
FrontFace, RightFace, FrontFace, -RightFace, TopFace, 

NonCommutativeMultiply @@
{cool /. Thread[{BottomFace, TopFace, FrontFace, BackFace, LeftFace, RightFace} 
  -> 
  Generators] /. -x_ -> Inverse[x]}

M[[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17, 
45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43, 
37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48],
{RightFace, -BackFace, -TopFace, -RightFace, TopFace, BackFace, -RightFace, FrontFace, 
LeftFace, -TopFace, -LeftFace, -BackFace, -LeftFace, BackFace, -FrontFace, TopFace, 
-RightFace, -BackFace, TopFace, BackFace, RightFace, RightFace, RightFace, -BackFace, 
-TopFace, -FrontFace, -RightFace, TopFace, RightFace, FrontFace, RightFace, FrontFace, 
-RightFace, TopFace, BackFace, -RightFace, -BackFace, -TopFace, -RightFace, 
-FrontFace, -TopFace, FrontFace, TopFace, RightFace, -FrontFace, LeftFace, TopFace, 
FrontFace, -TopFace, -FrontFace, FrontFace, -RightFace, -FrontFace, 
TopFace, FrontFace, RightFace, -FrontFace, -RightFace, -TopFace, -FrontFace, 
FrontFace, RightFace, -TopFace, -TopFace, BackFace, RightFace, -BackFace, -RightFace, 
-TopFace, RightFace, -TopFace, -FrontFace, -FrontFace, RightFace, FrontFace, 
RightFace, RightFace, -TopFace, -TopFace, -TopFace, TopFace, RightFace, TopFace, RightFace, 
FrontFace, RightFace, FrontFace, -RightFace, TopFace, 

Generators[[1]]
M[[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17, 45, 2, 
20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43, 37, 38, 39, 
40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48], {BottomFace}, 1]

%[[1]] = %[[[1]]]

True
\end{verbatim}