Non Commutative Gaussian

Elimination - Program 3

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc/SchreierSimsRubik/.

Pensieve Header: NCGE Program 3 - replacing tricks with better ones when possible, recursively. The results are pathetic.
The Cube

The Generating Permutations

\[ n = 54; \ $\text{RecursionLimit} = 2^{16}; \]
**Generators** = 

\[
\begin{align*}
M[\{18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17, \\
45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43, \\
37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48\}, \\
\{\text{BottomFace}\}, 1], \\
M[\{1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17, \\
18, 19, 20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36, \\
12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54\}, \\
\{\text{TopFace}\}, 1], \\
M[\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, \\
18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46, \\
39, 42, 45, 38, 41, 44, 37, 40, 43, 30, 29, 28, 49, 50, 51, 52, 53, 54\}, \\
\{\text{FrontFace}\}, 1], \\
M[\{3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14, \\
15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, \\
37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16\}, \\
\{\text{BackFace}\}, 1], \\
M[\{13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16, \\
17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35, \\
36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 47, 48, 49, 50, 51, 7, 53, 54\}, \\
\{\text{LeftFace}\}, 1], \\
M[\{1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27, \\
36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34, \\
37, 38, 15, 40, 41, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45\}, \\
\{\text{RightFace}\}, 1] \\
\};
Program 3

Clear[{s, M, T}; TC = 0;
M :> M[a1___, {w1___}, m1___] := M[a1[[a2]], {w1, w2}, m1 + m2];
M :> Inverse[M[a___, w___, m___]] := M[Ordering[a], -Reverse[w, m];
Feed[M[Range[n], __]] := Null;
Feed[M[a___, {w___}, m___]] := Module[
  modified = False, {i, j, w1, w2, m1, skl},
  For[i = 1, a[[i]] == 1, ++i]; j = a[[i]];
If[Head[sij = s[i, j]] === Integer,
  (* then *) If[m = TC[[3]] || False,
Feed[ReplacePart[Inverse[T[sij]] ** M[a, {w}, m], {-sij, w}, 2]],
  modified = True; T[s[i, j] = TC] = M[a, {w}, m];
Feed[ReplacePart[Inverse[M[a, {w}, m]], -{w, -sij}, 2]]
  ];
  (* else *) modified = True; T[s[i, j] = TC] = M[a, {w}, m]
];
If[modified,
  sij = s[i, j]; Do[
  If[Head[skl = s[k, l]] == Integer,
  Feed[ReplacePart[T[sij] ** T[skl], {sij, skl}, 2]],
  Feed[ReplacePart[T[skl] ** T[sij], {skl, sij}, 2]]
  ],
  {k, n}, {l, n}
  ];
];
Images[i___] := Prepend[Select[Range[n], Head[s[i, #]] === Integer &], i];
MoveCount[i___, i___] := 0;
MoveCount[i___, j___] := T[s[i, j]][[3]]
Dynamic[TC, Images /@ Range[n], Sum[Total[MoveCount[i, #] & /@ Images[i]], {i, n}]],
UpdateInterval → 1]

{126328,
{1, 3, 7, 9, 10, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 52, 54},
{2, 8, 19, 21, 25, 27, 38, 40, 42, 44},
{3, 7, 9, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 54},
{4}, {5}, {6}, {7, 9, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48},
{8, 19, 21, 25, 27, 38, 40, 42, 44},
{9, 15, 16, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48}, {10}, {11}, {12},
{13}, {14}, {15}, {16}, {17}, {18}, {19}, {21, 25, 27, 38, 40, 42, 44},
{27, 38, 40, 42, 44}, {28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48},
{29, 32, 35, 47},
{30, 31, 33, 34, 36, 37, 39, 45, 48}, {31}, {32, 35, 47}, {33, 34, 36, 39, 45, 48},
{34}, {35}, {36}, {37}, {38}, {39}, {40}, {41}, {42}, {43}, {44}, {45},
{46}, {47}, {48}, {49}, {50}, {51}, {52}, {53}, {54}, 156105796449}

http://dorbn.net/AcademicPensieve/2009-07/#MathematicaNotebooks
The Order of the Group

Timing[
  Feed[=]; Product[Length[Images[i]], {i, n}] & @ Generators
]

(Aborted after about 18 hours).