Non Commutative Gaussian Elimination - Program 3

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc/SchreierSimsRubik/.

Pensieve Header: NCGE Program 3 - replacing tricks with better ones when possible, recursively. The results are pathetic.

The Cube

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http://drorbn.net/AcademicPensieve/2009-07/#MathematicaNotebooks
The Generating Permutations

\begin{verbatim}
\textbf{n} = 54; \$\text{RecursionLimit} = 2^{16};
\textbf{Generators} = {
    \textbf{M}[[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17,
        45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43,
        37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48],
        \{\text{BottomFace}\}, 1],
    \textbf{M}[[1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17,
        18, 19, 20, 8, 14, 23, 32, 28, 27, 29, 7, 13, 22, 31, 37, 35, 36,
        12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54],
        \{\text{TopFace}\}, 1],
    \textbf{M}[[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
        18, 19, 20, 11, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 39, 42,
        45, 38, 41, 44, 37, 40, 43, 46, 30, 29, 28, 49, 50, 51, 52, 53, 54],
        \{\text{FrontFace}\}, 1],
    \textbf{M}[[3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14,
        15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36,
        37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16],
        \{\text{BackFace}\}, 1],
    \textbf{M}[[13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16,
        17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35,
        36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 49, 50, 51, 7, 53, 54],
        \{\text{LeftFace}\}, 1],
    \textbf{M}[[1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27,
        36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34,
        37, 38, 15, 40, 41, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45],
        \{\text{RightFace}\}, 1]
};
\end{verbatim}

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Program 3

Clear[s, M, T]; TC = 0;
M /: M[a_, {w___}, m_] ** M[a2_, {w2___}, m2_] := M[a1[[a2]], {w1, w2}, m1 + m2];
M /: Inverse[M[a_, w_, m_]] := M[Ordering[a], -Reverse[w], m];
Feed[Range[n], ___] := Null;
Feed[M[a_, {w___}, m_]] := Module[
{modified = False, i, j, sij, m, skl},
For[i = 1, a[[i]] = i, ++i; j = a[[i]];
If[Head[sij = s[i, j]] === Integer,
(* then *) If[m == T[sij][[3]] || False,
Feed[ReplacePart[Inverse[T[sij]] ** M[a, {w}, m], {-sij, w}, 2]],
modified = True; T[s[i, j] = ++TC] = M[a, {w}, m];
Feed[ReplacePart[Inverse[M[a, {w}, m]] ** T[sij], {-w, -sij}, 2]]
],
(* else *) modified = True; T[s[i, j] = ++TC] = M[a, {w}, m]
]
);
If[modified,
sij = s[i, j]; Do[
If[Head[skl = s[[k, l]]] === Integer,
Feed[ReplacePart[T[sij] ** T[skl], {sij, skl}, 2]]; Feed[ReplacePart[T[skl] ** T[sij], {skl, sij}, 2]]
], {k, n}, {l, n}
]
];
Images[i_, ___] := Prepend[Select[Range[n], Head[s[i, #]] === Integer &], i];
MoveCount[i_, i_, ___] := 0; MoveCount[i_, j_, ___] := T[s[i, j]][[3]]; Dynamic[{TC, Images /@ Range[n], Sum[Total[MoveCount[i, #] & /@ Images[i], {i, n}]],
UpdateInterval -> 1]

{126328,
{1, 3, 7, 9, 10, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 52, 54},
{2, 8, 19, 21, 25, 27, 38, 40, 44},
{3, 7, 9, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 54},
{4}, {5}, {6}, {7, 9, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48},
{8, 19, 21, 25, 27, 38, 40, 42, 44},
{9, 15, 16, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48}, {10}, {11}, {12},
{13}, {14}, {15}, {16}, {17}, {18}, {19, 21, 25, 27, 38, 40, 42, 44}, {20},
{21, 25, 27, 38, 40, 42, 44}, {22}, {23}, {24, 29, 32, 35, 47, 51}, {25}, {26},
{27, 38, 40, 42, 44}, {28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48}, {29, 32, 35, 47},
{30, 31, 33, 34, 36, 37, 39, 45, 48}, {31}, {32, 35, 47}, {33, 34, 36, 39, 45, 48},
{34}, {35}, {36}, {37}, {38}, {39}, {40}, {41}, {42}, {43}, {44}, {45},
{46}, {47}, {48}, {49}, {50}, {51}, {52}, {53}, {54}, {156105796449}

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The Order of the Group

Timing[
  (Feed[#]; Product[Length[Images[i]], {i, n}] & /@ Generators
]

(Aborted after about 18 hours).