Non Commutative Gaussian
Elimination - Program 0

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc/SchreierSimsRubik/.


Program 0

\[
gs = \{\text{purple} = P[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43, 37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48], \\
\text{white} = P[1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17, 18, 19, 20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36, 12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54], \\
\text{green} = P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46, 49, 42, 45, 38, 41, 44, 37, 40, 43, 30, 28, 49, 50, 51, 52, 53, 54], \\
\text{blue} = P[3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16], \\
\text{red} = P[13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16, 17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35, 36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 4, 50, 51, 7, 53, 54], \\
\text{yellow} = P[1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27, 36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34, 37, 38, 15, 40, 41, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45])\};
\]
$\text{RecursionLimit} = 2^{16};$

\begin{verbatim}
 n = 54;
 PY := p \[\_\_\_] = p[[\_\_]]; 
Inv[p\_] := p \[Ordering[p]\];
Feed[p\@Range[n]] := Null;
Feed[p\_] := Module[{i, j},
  For[i = 1, p[[i]] == i, ++i; j = p[[i]]; 
  If[Head[s[i, j]] === p, 
  Feed[Inv[s[i, j]] ** p], 
  (*Else*) s[i, j] = p; 
  Do[If[Head[s[k, l]] === p, 
    Feed[s[i, j] ** s[k, l]]; 
    Feed[s[k, l] ** s[i, j]] 
  ], 
  {k, n}, {l, n}]
  ]]
];
(Feed[\_]; Product[1 + Length[Select[Range[n], Head[s[i, \_\_] === p \&]], {i, n}]) 
/@ gs
\end{verbatim}

{4, 16, 159993501696000, 2111914223872000, 43252003274489856000, 43252003274489856000}
Images[i_] := (i) & Select[Range[n], Head[s[i]] === P &];
ListPlot[
  Join @@ Table[{i, n} & @ Images[i], {i, n}],
  AspectRatio -> 1
]