Question from Wigderson

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6:32 PM

Date: Sat, 13 Jun 2009 18:25:07 -0400
From: Avi Wigderson <avi@ias.edu>
To: Dror Bar-Natan <drorbn@math.toronto.edu>
Subject: Re: MathCamp

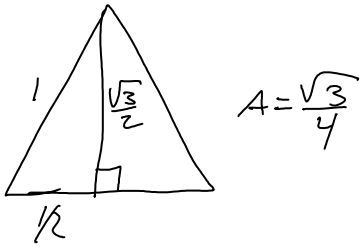
PS I noticed you (or your homepage) like tiles.
I was recently involved with the following, on which you can test yourgeometric intuition:
you want to tile R^d periodically according to the lattice Z^d. What is theminimum surface area of the tile.
It is certainly between that of the (unit volume) sphere and cube, namely (up
to constants) between \sqrt{d} and d.
What do you think is the (asymptotic) truth?

Ciao, Avi

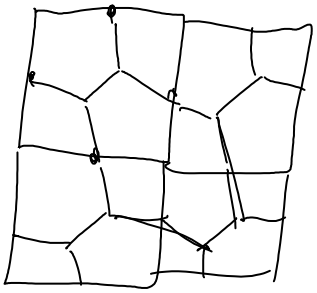
The sphere in $\mathbb{R}^d$

$V = S r/d = C_d r^d = 1$

$\Rightarrow S = d/r$



$A = \frac{\sqrt{3}}{4}$



$=: H$

$A = \frac{3\sqrt{3}}{2}$

$p = 6$

$\frac{p}{\sqrt{A}} = \frac{6\sqrt{2}}{3^{3/4}} =$

$= 2^{3/2} 3^{1/4} \sim 3.72 = 4$

$H^{d/2}$ gives $\frac{4}{2}d < \frac{4}{2}d$